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Solutions

1. Decimals, Fractions and Percentages

Decimals

1. Evaluate $8.1 - 19.4 \div 4$ 2 KU
2. Evaluate $43 - 5.6 \times 4$ 2 KU
3. Evaluate $5.7 + 3.9 \times 4$ 2 KU
4. Evaluate $31.4 - 27.09 \div 3$ 2 KU

Fractions

5. Evaluate $4\frac{5}{6} + 2\frac{3}{5}$ 2 KU
6. Evaluate $4\frac{2}{5} - 1\frac{2}{3}$ 2 KU
7. Evaluate $2\frac{3}{4} \times 1\frac{1}{3}$ 2 KU
8. Evaluate $5\frac{1}{2} \div 1\frac{3}{8}$ 2 KU
9. Evaluate: $\frac{3}{8}$ of $(1\frac{2}{3} - \frac{4}{7})$. 2 KU
10. Evaluate $\frac{3}{7}(1\frac{5}{6} + \frac{3}{4})$ 2 KU

Various

11. Evaluate $23 + (-6)^2 \times \frac{3}{4}$ 2 KU
12. Evaluate 32% of £850 2 KU
13. Find $\frac{3}{8}$ of 544 2 KU

Using Percentages

1. Bacteria in a test tube increase at the rate of 0.9% per hour.
At 12 noon there are 4500 bacteria.
At 3 pm, how many bacteria will be present?
Give your answer **to 3 significant figures**. 4 KU
2. In January 2001, it was estimated that the number of flamingos in a colony was 7000.
The number of flamingos is decreasing at the rate of 14% per year.
How many flamingos are expected to be in this colony in January 2005 ?
Give your answer **to the nearest 10**. 4 KU
3. In 1999, a house was valued at £70,000 and the contents were valued at £45,000.
The value of the house **appreciates** by 7% each year.
The value of the contents **depreciates** by 9% each year.
What will be the **total** value of the house **and** contents in 2002 ? 3 KU
4. A factory was put on the market in January 2001.
The site was in an excellent location so the value of the building has appreciated since then by 5.3% per year.
Unfortunately the plant & machinery were poorly maintained and have depreciated by 8.5% per year.
The value of the building was £435 000 and the value of the plant & machinery was £156 000 in January 2001.
What would be the expected value of the complete factory in January 2003 ? 4 KU
5. How much would the Strachans pay for a new iron, priced £16.50 at Watsons ? 3 KU
- WATSON'S SALE
 $66\frac{2}{3}\%$ off everything
6. In 1995, the price of 1 litre of a certain kind of petrol was 54.9 pence
By 1996, the price of 1 litre of the same kind of petrol had risen to 56.3 pence.
The percentage increase for each of the next four years is expected to be the same as the percentage increase between 1995 and 1996.
What is the price of 1 litre of petrol expected to be in the year 2000? 4 RE

Reversing the change

7. A computer is sold for £695. This price includes VAT at 17.5%
Calculate the price of the computer **without** VAT. 3 KU
8. During the Christmas Sales a shopkeeper sold 60% of his "Santa Claus Dolls"
He then found he was left with 50 dolls.
How many dolls had he in stock to begin with ? 3 KU
9. Kerry bought a new car in 1996. When she sold it four years later, she found that it had reduced in value by 60% and she received only £4640.
How much had Kerry paid for the car in 1996 ? 3 KU
10. James bought a car last year. It has lost $12\frac{1}{2}\%$ of its value since then.
It is now valued at £14 875.
How much did James pay for his car. 2 KU

Standard Form

1. Each of these large oil containers holds 4.80×10^8 litres of the fuel. How many litres are there altogether in the full tanks shown? Give your answer in scientific notation.



2 KU

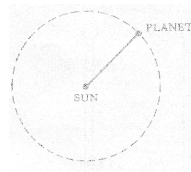
2. A newspaper report stated "Concorde has now flown 7.1×10^7 miles. This is equivalent to 300 journeys from the earth to the moon." Calculate the distance from the earth to the moon. Give your answer in **scientific notation correct to 2 significant figures.**

3 KU

3. The planet Mars is at a distance of 2.3×10^8 kilometres from the Sun. The speed of light is 3.0×10^5 km per second. How long does it take light from the Sun to reach Mars? Give your answer to the nearest minute.

3 KU

4. A planet takes 88 days to travel round the Sun. The approximate path of the planet round the Sun is a circle with diameter 1.2×10^7 kilometres. Find the speed of the planet as it travels round the Sun.



Give your answer in kilometres per hour, correct to 2 significant figures.

4 KU

5. The mass of a proton is approximately 1.8×10^3 times greater than the mass of an electron. If the mass of an electron is 9.11×10^{-31} kg, calculate the mass of a proton. Give your answer in **scientific notation correct to 2 significant figures.**

2 KU

6. Large distances in space are measured in light years. A camera on a space telescope, photographs a galaxy, a distance of 50 million light years away. One light year is approximately 9.46×10^{12} kilometres. Calculate the distance of the galaxy from the space telescope in kilometres. Give your answer in **scientific notation**

2 KU

7. The annual profit (£) of a company was 3.2×10^9 for the year 1997. What profit did the company make per second. Give your answer to **three significant figures.**

2 KU

8. The total number of visitors to an exhibition was 2.925×10^7 . The exhibition was open each day from 5 June to 20 September **inclusive**. Calculate the average number of visitors per day to the exhibition.

3 KU

9. The mass of the sun is 2.2×10^{30} kilograms. The mass of the earth is 5.97×10^{24} kilograms. Express the mass of the earth as a percentage of the mass of the sun. Give your answer in **scientific notation.**

3 KU

2. Algebra 1 – Basic Algebraic operations, Indices and Surds

Evaluation

1. Evaluate $30 - 3p^2q$ where $p = -1$ and $q = -6$ 2 KU

Simplification

2. Simplify $4(3x - 2) - 5(4x + 1)$ 3 KU
3. Remove the brackets and collect like terms $(3a - b)(2a - 5b)$ 2 KU
4. Remove the brackets and simplify your answer $(2x - 1)(x + 3) + (x - 4)^2$ 4 KU
5. Remove the brackets and simplify $(3y - 4)^2$ 2 KU
6. Multiply out the brackets and simplify. $(2x - 3)(3x^2 + 4x - 1)$ 3 KU

Factorisation

7. Factorise $6x^2 - 9x$ 2 KU
8. Factorise $4a^2 - 9b^2$ 2 KU
9. a) Factorise the expression $9x^2 - y^2$ 1 KU
b) Hence simplify $\frac{6x + 2y}{9x^2 - y^2}$ 2 KU
10. a) Factorise $a^2 - 9b^2$ 1 KU
b) Hence simplify $\frac{a^2 - 9b^2}{2a + 6b}$ 2 KU
11. a) Factorise $x^2 - 9$ 1 KU
b) Express $\frac{4(5x + 3)}{25x^2 - 9}$ in its simplest form 2 KU
12. Express $\frac{15x - 20}{9x^2 - 16}$ in its simplest form 3 KU
13. i) Factorise **completely** $2x^2 - 6x$ 1 KU
ii) Express $\frac{2x^2 - 6x}{x^2 - 9}$ in its simplest form. 2 KU
14. Factorise $3x^2 - 13x - 10$ 2 KU

Solve Linear Equations

15. Solve the equation $5 - 2(1 + 3x) = 27$ 3 KU
16. Solve the equation $5 + 3a = a - 15$ 3 KU

Simultaneous Equations

17. Solve **algebraically**, the system of equations $2a + 4b = -7$
 $3a - 5b = 17$ 3 KU
18. Solve the system of equations $5a + 3b = 9$
 $7a - 2b = 25$ 3 KU

Functions

1. $f(x) = x^2 - 2x$, evaluate $f(-2)$ 2 KU
2. $h(t) = 15t - 3t^2$ Find $h(-2)$ 2 KU
3. Given that $f(x) = \frac{x^3 + x^2 + 2}{5x - 1}$ evaluate $f(-3)$ 3 KU
4. $f(x) = 9 - 6x$
(a) Evaluate $f(-3)$ 1 KU
(b) Given that $f(t) = 11$, find t 2 KU
5. The function $f(x)$ is given by the formula $f(x) = 3x^2 - 7$, where x is a real number.
(a) Find the value of $f(-2)$. 2 KU
(b) Find the **values** of a for which $f(a) = 20$. 3 KU
6. $f(x) = \frac{4}{x^2}$ find $f\left(\frac{1}{2}\right)$ 2 KU
7. $f(x) = 3^x$
a) Find $f(4)$ 1 KU
b) Given that $f(x) = \sqrt{27}$, find x . 3 KU
8. $f(x) = \frac{3}{\sqrt{x}}$ Find the **exact** value of $f(2)$
Give your answer **as a fraction** with a rational denominator. 2 KU
9. $f(x) = 3\sqrt{x}$ Find the exact value of $f(12)$,
giving your answer as a **surd, in its simplest form.** 2 KU

Quadratic Equations

1. Solve **algebraically**, the equation $x^2 = 7x$ 3 KU
2. Solve **algebraically**, the equation $6y - y^2 = 0$ 2 KU
3. Solve **algebraically**, the equation $2x^2 - 9x - 5 = 0$ 3 KU
4. Solve for x : $2x^2 + 7x - 15 = 0$ 3 KU
5. Solve the equation $2x^2 + 5x - 12 = 0$ 3 KU
6. Solve the equation $2p^2 - p - 10 = 0$ where p is a real number. 3 KU
7. Two functions are given below:
$$f(x) = x^2 + 2x - 1$$
$$g(x) = 5x + 3$$

Find the values of x for which $f(x) = g(x)$ 3 KU
8. Find the two roots of the equation $2x^2 - 3x - 4 = 0$
(Answer correct to 1 decimal place). 4 KU
9. Solve the equation $x^2 + 2x - 6 = 0$
Give your answers correct to 2 significant figures. 5 KU

Inequalities

1. Solve the inequality $8 - x > 3(2x + 5)$ 3 KU
2. Solve **algebraically** the inequality $3y < 4 - (y + 2)$ 3 KU
3. Solve the inequality $3 - (x - 6) < 2x$ 3 KU
4. Solve algebraically the inequality $6x - 2 < 5(1 - 3x)$ 3 KU
5. Solve algebraically, the inequality $2 + 5x \geq 8x - 16$ 3 KU
6. Solve the inequality $2 - 5(3x - 2) \geq 4(1 - 3x)$ where x is a **positive integer**. 5 KU
7. An inequality, like $4x + 10 \leq 6x + 2 \leq 3x + 26$, can be solved by
 - i) solving $4x + 10 \leq 6x + 2$ and
solving $6x + 2 \leq 3x + 26$
 - ii) looking carefully at the two sets of answers to decide
on the correct solution to the original inequality.
 - a) Solve $3x + 1 \leq 5x + 3 \leq x + 23$ 4 KU
 - b) Write down the set of **all** possible solutions where x is an INTEGER. 1 KU

Changing the subject of the formula

1. $Y = \frac{3(2v-w)}{5}$ Change the subject of the formula to v . 3 KU

2. $P = \frac{1}{3}(m-s)$ Change the subject of the formula to m 2 KU

3. $L = 8 + \frac{6}{Y}$ Change the subject of the formula to Y . 3 KU

4. Change the subject of the formula to k . $d = \frac{k-m}{t}$ 2 KU

5. $Q = p^2 + 3T$ Change the subject of the formula to T . 2 KU

6. $M = R^2t - 3$ Change the subject of the formula to R . 3 KU

7. Change the subject of the formula to b . $A = \sqrt{4b^2 - c}$ 3 KU

8. a) Change the subject of the formula $Q = 2\sqrt{s} + t$, to s 3 KU

b) Find the value of s when $Q = 3.5$ and $t = 2.2$ 2 KU

9. The frequency, F hertz of the sound you hear as you drive past a factory siren at a speed of v metres per second is given by the formula

$$F = f \left(1 - \frac{v}{s} \right)$$

where f is the true frequency of the sound emitted by the siren and s is the speed of sound. Change the subject of the above formula to v .

3 KU

Algebraic Fractions

1. Express as a single fraction in its simplest form $\frac{1}{2x} - \frac{1}{3x}$, $x \neq 0$ 2 KU

2. Express as a single fraction in its simplest form $\frac{3}{x} + \frac{2-x}{x^2}$, $x \neq 0$ 3 KU

3. Express as a single fraction in its simplest form $\frac{5}{x} - \frac{3}{(x-2)}$, $x \neq 0$ or $x \neq 2$ 3 KU

Fraction Equations

1. Solve the equation $\frac{2x+1}{3} - \frac{x}{4} = 2$ 3 KU

2. Solve the equation $\frac{x+4}{2} - \frac{2x+1}{3} = 1$, where x is a real number. 3 KU

3. Solve **algebraically** the equation $3x - \frac{(5x+2)}{4} = 3$ 3 KU

4. Solve the equation $\frac{x-3}{2} + \frac{2x-1}{3} = 4$ 4 KU

5. Solve this equation for x : $\frac{x-2}{3} - \frac{x}{2} = \frac{1}{4}$ 4 KU

6. Solve **algebraically**, the equation $\frac{x}{2} - \frac{(x+1)}{3} = 4$ 3 KU

7. Solve **algebraically**, the equation $\frac{m}{3} = \frac{(1-m)}{5}$ 3 KU

Indices

1. Evaluate $27^{\frac{2}{3}}$ 2 KU
2. Express in its simplest form $y^{10} \times (y^4)^{-2}$ 2 KU
3. Simplify $a^3(a^{-7} + 5)$ 2 KU
4. Express $\frac{3y^5 \times 4y^{-1}}{6y}$ in its simplest form. 3 KU
5. Express $\frac{y^4 \times y}{y^{-2}}$ in its simplest form. 2 KU
6. Express $\frac{b^{\frac{1}{2}} \times b^{\frac{3}{2}}}{b}$ in its simplest form. 2 KU
7. Remove the brackets and simplify $b^{\frac{1}{2}} \left(b^{\frac{1}{2}} + b^{-\frac{1}{2}} \right)$ 3 KU
8. Remove the brackets and simplify $a^{\frac{1}{2}} \left(a + \frac{1}{a} \right)$ 2 KU

Surds

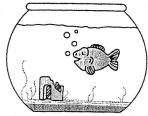
- Express $\sqrt{50}$ as a surd in its simplest form. 1 KU
- Simplify $\frac{\sqrt{72}}{\sqrt{3}}$ 2 KU
- Simplify $\sqrt{48} - 3\sqrt{3}$ 2 KU
- Express $\sqrt{32} - \sqrt{2}$ as a surd in its simplest form. 2 KU
- Express $\sqrt{72} - \sqrt{2} + \sqrt{50}$ as a surd in its simplest form 3 KU
- Express $\sqrt{32} + \sqrt{8}$ as a surd in its simplest form. 3 KU
- Multiply out the brackets $\sqrt{2}(\sqrt{6} - \sqrt{2})$
Express your answer as a **surd** in its simplest form. 2 KU
- $f(x) = 3\sqrt{x}$
Find the exact value of $f(12)$, giving your answer as a **surd, in its simplest form.** 2 KU
- Express $\frac{3}{\sqrt{5}}$ as a fraction with a rational denominator. 2 KU
- Simplify $\frac{\sqrt{3}}{\sqrt{24}}$ Express your answer as a fraction with a rational denominator 3 KU
- $f(x) = \frac{3}{\sqrt{x}}$ Find the **exact** value of $f(2)$
Give your answer **as a fraction** with a rational denominator. 2 KU
- A function f is given by $f(x) = 4^x$
Find the value of $f\left(\frac{3}{2}\right)$ 2 KU

3. Data Handling

Simple Probability - **Note:** You should always give your answer in its **simplest form**

(Questions 3 to 8 in this section are not from Past Papers – but you should know how to do them.)

1. A bag contains **red, green, blue, yellow** and **white** balls.
There are 10 of **each** colour, numbered from 1 to 10.
The balls are placed in a drum and one is drawn out.
 - a) What is the probability that it is a **7** ? 1 KU
 - b) What is the probability it is a **blue 7** ? 1 KU

2. Roy and Zara go to the fairground.
A stall has a card game where a goldfish can be won if anyone can turn over a face card from a pack of 52 cards which are placed face down.

Calculate the probability, in its simplest form, of Zara winning the goldfish. 3 KU

3. A box contains **5 red, 6 green, 7 blue and 2 yellow** coloured pencils.
Jenny picks one out of the box
 - a) What is the probability that it is a **green pencil** 1
 - b) She does **NOT** replace the pencil, but draws another one
What is the probability that this is a blue pencil 2

4. A bag contains **10 red, 25 green, 9 blue and 6 yellow** marbles.
Sam picks one out of the bag, **replaces** it and then picks another one.
What is the probability that he picked a **Green** marble followed by a **Red** one 3

5. Michelle estimates that the probability that her hockey team will win their next game is 0.2, and the probability they will draw is 0.5
 - a) Calculate P(Win or Draw) 1
 - b) Calculate P(Lose) 1

6. Robin is the member of an archery club. On average 80% of his shots hit the target.
What is the probability that:
 - a) He misses the target 1
 - b) He hits the target 3 times in a row 1
 - c) He hits the target with the first shot, and misses with the next two shots. 1

7. When microprocessors are made, it is known that in any batch, 15% are defective.
 - a) What is the probability of picking a microprocessor that is **NOT** defective 1
 - b) A batch of 5000 microprocessors are produced. How many would be expected to have **NO** defects. 2

8. Three new students are about to join a class. Assuming that $P(\text{male}) = \frac{1}{2}$
 - a) What is the probability that all three will be boys ? 1
 - b) If you are told that one is a boy, what is the probability now, that all three will be boys. 2

Probability from relative Frequency

1. A garage carried out a survey on 600 cars. The results are shown in the table below:

		Engine size (cc)			
		0 – 1000	1001 – 1500	1501 – 2000	2001 +
Age	Less than 3 years	50	80	160	20
	3 years or more	60	100	120	10

- a) What is the probability that a car chosen at random, is less than 3 years old? 1 KU
- b) In a sample of 4200 cars, how many would be expected to have an engine size greater than 2000cc **and** be 3 or more years old. 2 KU
2. The National Tourist Association carried out a survey amongst 500 adults from the UK to find out what would influence them most when choosing a holiday.

The results of the survey are shown in the table below.

Age	Price	Weather	Facilities	Scenery
35 and under	190	65	23	7
Over 35	95	35	12	73

- a) What is the probability that any adult chosen at random would have scenery as their main priority when choosing a holiday? 1 KU
- b) A 25 year old adult is chosen at random. What is the probability that the facilities is his/her main concern when choosing a holiday? 2 KU
- c) What is the probability that any adult chosen at random **will not** have cost as their main concern when choosing a holiday? 2 KU

3. A group of people who admitted to drinking bottled water were asked if they preferred FIZZY water or STILL water.

The results are shown in this table.

What is the probability that:

	FIZZY	STILL
aged 10 to 20	65	5
aged over 20	10	30

- a) a person chosen at random **from this sample** will prefer STILL water. 1 KU
- b) the person chosen will be over 20 years old **and** prefer FIZZY water. 2 KU
- Note: to gain full credit in this question, both answers must be in their simplest form.*

4. Smiley's Garage was asked to supply information on last month's sales. They were asked to identify the number of used and new cars purchased. The results are shown in the table.

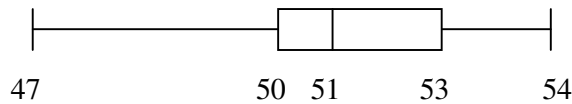
	new car	used car
aged 18 to 40	17	30
aged over 40	23	50

What is the probability that a person chosen at random **from this sample** will

- a) have bought a new car? 1 KU
- b) be between 18 and 40 years old **and** have bought a used car? 1 KU

Statistical Diagrams

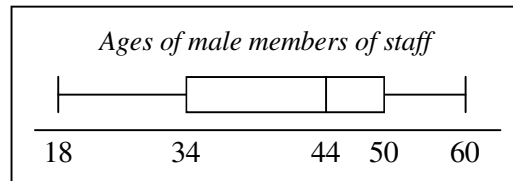
1. A random check is carried out on the contents of a number of matchboxes. A summary of the results is shown in the boxplot below.



What percentage of matchboxes contains fewer than 50 matches.

1 RE

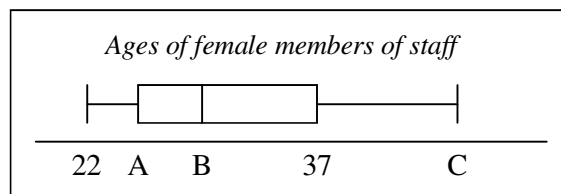
2. The ages of the male members of staff in a school were recorded and a box plot was drawn to show the results.



When the same study was carried out for the female members of staff in the same school, another box plot was drawn.

It was found that:

- the **range** of the ladies' ages was **half** that of the range of the mens'.
- the ladies' **median** age was **15 years less** than the men's median age.
- the **semi-interquartile range** of the ladies' was **three quarters** that of the men's.



Make a copy of the above females' box plot and complete it to show which ages are represented by the letters **A**, **B** and **C**.

3 RE

3. Fifteen medical centres each handed out a questionnaire to fifty patients. The numbers who replied to each centre are shown below.

11	19	22	25	25
29	31	34	36	38
40	46	49	50	50

Also, they each posted the questionnaires to another fifty patients. The numbers who replied to each centre are shown below.

15	15	21	22	23
25	26	31	33	34
37	39	41	46	46

Draw an appropriate statistical diagram to compare these two sets of data.

3 RE

4. A furniture maker investigates the delivery times, in days, of two local wood companies and obtains the following data.

<i>Company</i>	<i>Minimum</i>	<i>Maximum</i>	<i>Lower Quartile</i>	<i>Median</i>	<i>Upper Quartile</i>
Timberplan	16	56	34	38	45
Allwoods	18	53	22	36	49

- a) Draw an appropriate statistical diagram to illustrate these two sets of data. 3 RE
- b) Given that consistency of delivery is the most important factor, which company should the furniture maker use? Give a reason for your answer. 1 RE

5. Jamie conducted a survey.
He asked his classmates how they had travelled to school that day.
Here are their replies.

Walk 13
Bus 9
Car 6
Cycle 2

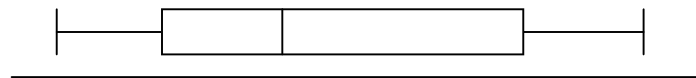
Draw an appropriate statistical diagram to illustrate this information 4 RE

6. The stem and leaf diagram shows a sample of 50 scores in a boy's golf tournament.

Golf Scores	
6	3 4
6	5 5 5 6 6 6 6 7 7 8 9 9 9
7	0 0 0 0 1 1 2 3 3
7	5 6 6 6 7 8
8	0 0 1 1 2 2 3
8	5 6 6 6 7 8 9
9	2 3 4
9	7 8

- a) Write down the **median** golf score. 1 KU
- b) Calculate the **semi-interquartile range** for these scores. 3 KU
- c) Sketch this boxplot and fill in the correct values to illustrate the golf scores in this sample.

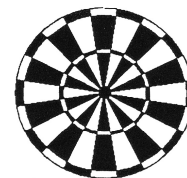
8 | 0 represents a score of 80



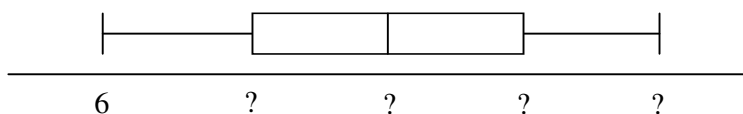
2 KU

7. In a tournament, 13 men throw one dart each at a dart board and their scores are noted.

Alex	16	Nick	20	Steve	28
Norrie	6	George	9	Brian	18
Ted	24	James	22	Graeme	18
Tom	12	John	13	Tony	7
George	9				



- a) Find the **median** and the upper and lower quartiles. 3 KU
- b) Make a neat sketch of the following box plot and fill in all the missing values. 2 KU



Standard Deviation

1. Fiona checks out the price of a litre of milk in several shops.

The prices in pence are:

49 44 41 52 47 43

- a) Find the mean price of a litre of milk. 1 KU
- b) Find the standard deviation of the prices. 2 KU
- c) Fiona also checks out the price of a kilogram of sugar in the same shops and finds that the standard deviation of the prices is 2.6. Make one valid comparison between the two sets of prices. 1 RE

2. A group of fifth year students from Alloa High School were asked how many hours studying they did in the week prior to their exams.

The results are shown below.

14 7 9 12 19 10 16 15

- (a) Use an appropriate formula to calculate the mean and standard deviation of these times. 3 KU
- (b) A similar group of students from Alloa Academy were asked the same question. The mean number of hours studied was 16 and the standard deviation was 2.2. How did the number of hours studied by students from Alloa High School compare with the number of hours studied by students from Alloa Academy? 2 RE

3. The Mobile Phone Shop is advertising their five latest mobile phones on their website.

Their prices are:

£120 £135 £75 £235 £185

Use an appropriate formula to calculate the mean and standard deviation of these prices.

(Show all working)



4 KU

4. The price, in pence per litre, of petrol at 10 city garages is shown below:

84.2 84.4 85.1 83.9 81.0

84.2 85.6 85.2 84.9 84.8

- a) Calculate the mean and standard deviation of these prices. 3 KU
- b) In 10 rural garages, the petrol prices had a mean of 88.8 and a standard deviation of 2.4. How do the rural prices compare with the city prices? 2 RE

5. Jim typed six pages on his computer using his word processor.

He did a "spell check" and discovered that he had made the following numbers of errors on the 6 pages.

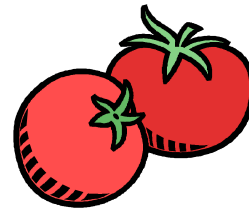
page one - 4 errors
page two - 1 errors
page three - 7 errors
page four - 13 errors
page five - 6 errors
page six - 5 errors

- a) Calculate the mean number of errors 1 KU
b) Calculate the standard deviation. 4 KU

6. After trying a new fertilizer on one of his tomato plants, a grower counted the number of tomatoes on each of its six bunches.

The number of tomatoes was:

8, 14, 9, 16, 13, 18

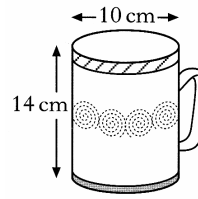


- a) Calculate the mean number of tomatoes. 1 KU
b) Construct a table and use it to calculate the standard deviation. 4 KU

4. Area & Volume

1. A mug is in the shape of a cylinder with diameter 10 centimetres and height 14 centimetres.

- a) Calculate the volume of the mug.
 b) 600 millilitres of coffee are poured in.
 Calculate the depth of the coffee in the cup.



2 KU

3 RE

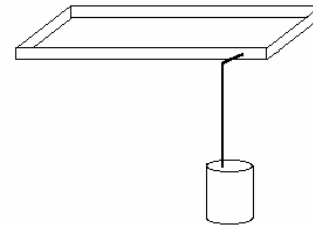
2. Rainwater is collected in a rectangular based tank on top of a flat roof and is drained periodically to a cylindrical tank on the ground where it is used for watering plants in dry weather.

The tank on the roof measures 3 metres by 9 metres and has a depth of 0.25 metres.

The tank on the ground is 1.85 metres high and has base radius of 0.55 metres.

Both tanks were empty, but after a heavy shower all the rainwater from the roof tank was drained to the ground tank and completely filled it.

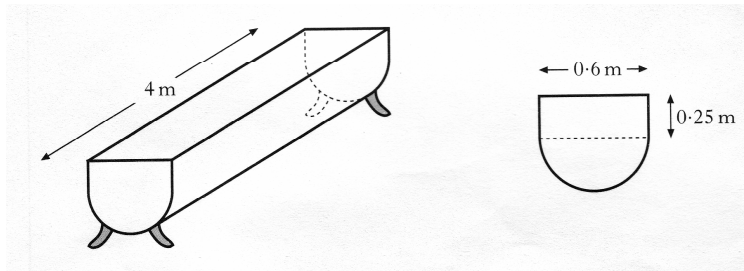
Calculate the depth of rainwater, to the nearest millimetre, in the roof tank immediately before it was drained to the ground tank.



5 RE

3. A feeding trough, 4 metres long, is prism shaped.

The uniform cross-section is made up of a rectangle and semi-circle as shown below.

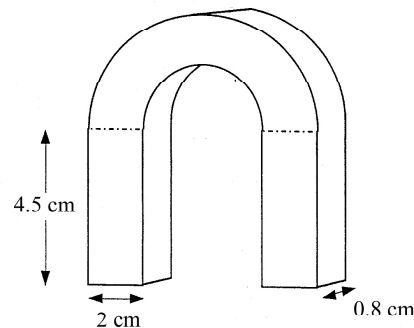
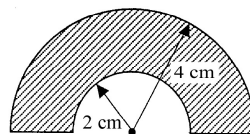


Find the volume of the trough, **correct to 2 significant figures**.

5 KU

4. The diagram shows a horse-shoe magnet.

The face of the arched part at the top consists of two semi-circles, with radii 2 centimetres and 4 centimetres.



Calculate the shaded area and use this to calculate the volume of metal required to make the magnet.

Give your answer correct to 1 decimal place.

5 RE

5. A cylindrical soft drinks can is 15 centimetres in height and 6.5 centimetres in diameter.
A new cylindrical can holds the same volume but has a reduced height of 12 centimetres.
What is the height of the new can ?

Give your answer **to 1 decimal place**.

4 RE

6. A metal doorstep is prism shaped,
as shown in Figure 1
The uniform cross-section
as shown in Figure 2:

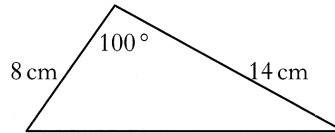


Figure 2

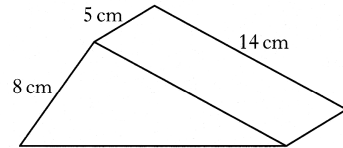


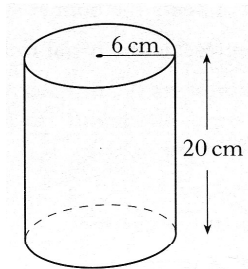
Figure 1

Find the volume of metal required to make the doorstep.

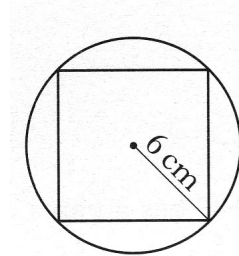
4 KU

7. A glass vase, in the shape of a cuboid with a square base is 20 centimetres high.

It is packed in a cardboard cylinder with radius 6 centimetres and height 20 centimetres.



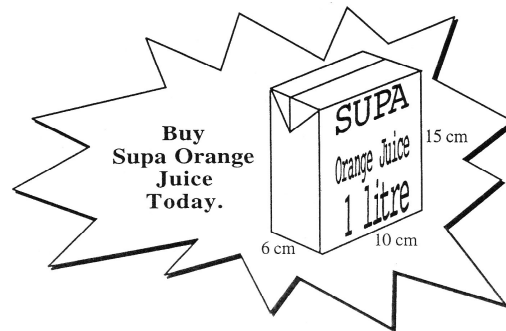
The corners of the vase touch the inside of the cylinder as shown.



Show that the volume of the space between the vase and the cylinder is $720(\pi - 2)$ cubic centimetres.

5 RE

8. a) Explain what is wrong with this advert for a 1 litre carton of Orange Juice.

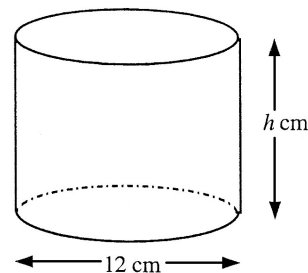


2 KU

- b) The measurements 10 cm, 6 cm and 15 cm are correct.

All of the juice is poured into this cylindrical container with base diameter 12 cm and it is found to exactly half fill it.

Calculate the height of the container.



4 RE

9. A wooden toy box is prism-shaped as shown in figure 1.

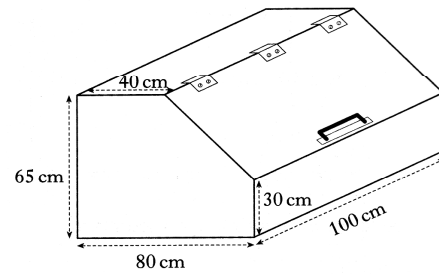


figure 1

The uniform cross-section of the box is as shown in figure 2.

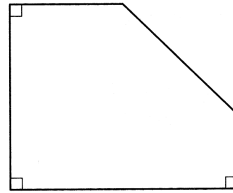


figure 2

Calculate the volume of the box in **cubic metres**.

4 KU

10. A skip is prism shaped as shown in figure 1.

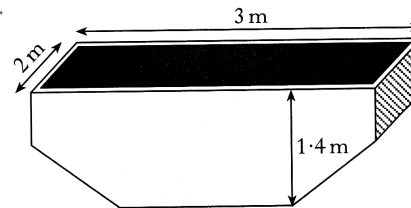


Figure 1.

The cross section of the skip, with measurements in metres, is shown in figure 2.

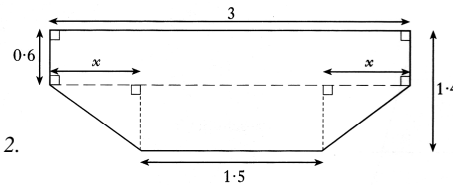


Figure 2.

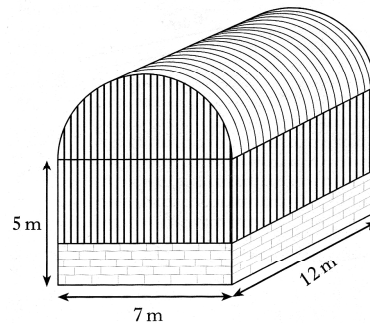
- Find the value of x .
- Find the volume of the skip in cubic metres.

1 KU

3 KU

11. A storage barn is prism shaped, as shown.

The cross-section of the storage barn consists of a rectangle measuring 7 metres by 5 metres and a semi-circle of radius 3.5 metres.



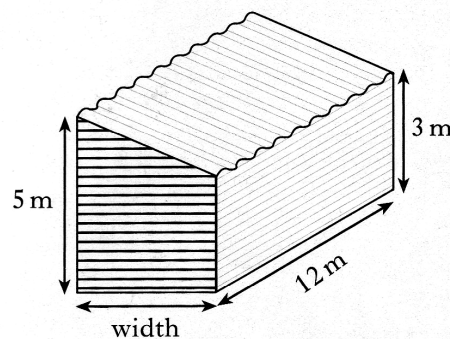
4 KU

- Find the volume of the storage barn.
Give your answers in cubic metres, **correct to 2 significant figures**.

- An extension to the barn is planned to increase the volume by 200 cubic metres.

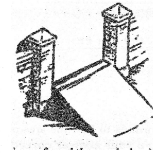
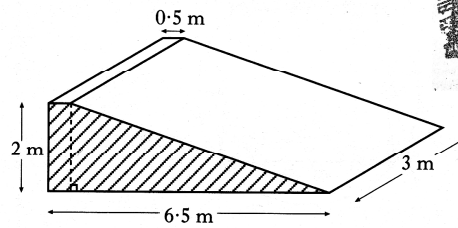
The uniform cross-section of the extension consists of a rectangle and a right angled triangle.

Find the width of the extension.



3 RE

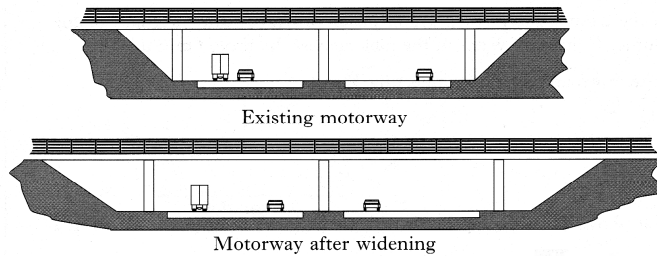
12. A ramp is being made from concrete.
The uniform cross section of the ramp consists of a right angled triangle and a rectangle as shaded in the diagram.



Find the volume of concrete required
To make the ramp.

2 KU

13. Ground has to be blasted and removed so that a motorway can be widened.
The existing motorway and the motorway after widening are shown below.



The uniform cross-section of the existing motorway consists of a rectangle and two congruent right angled triangles as shown in figure 1.

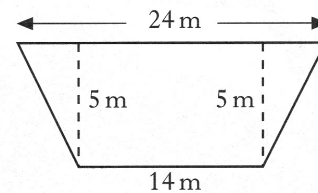


figure 1

The uniform cross-section of the motorway after widening consists of a rectangle and two congruent right angled triangles as shown in figure 2.

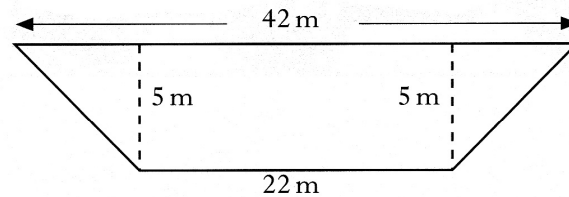


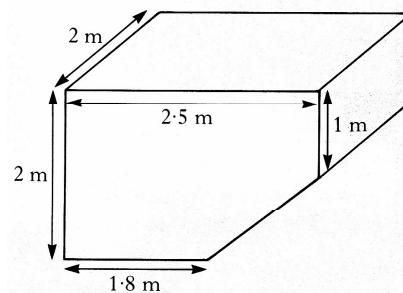
figure 2

The cost of blasting and removing each cubic metre of ground is £4.
10 kilometres of existing motorway is to be widened.
Find the total cost of blasting and removing the ground.

4 RE

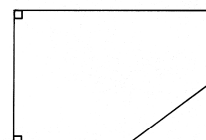
14. A bottle bank is prism shaped,
as shown in figure 1.

Figure 1



The uniform cross-section
is shown in figure 2.

Figure 2



Find the volume of the bottle bank.

4 KU

5. Similar Shapes and Similar Triangles

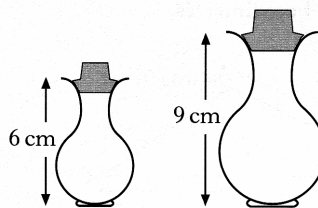
Similar Shapes – Area and Volume Scale Factors

1. Two perfume bottles are mathematically similar in shape.

The smaller one is 6 centimetres high and holds 30 millilitres of perfume.

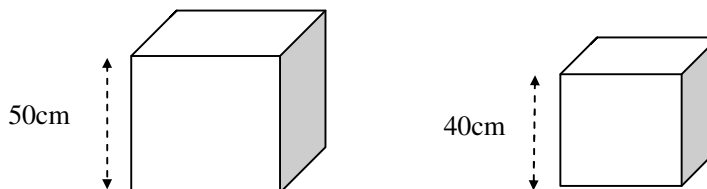
The larger one is 9 centimetres high.

What volume of perfume will the larger one hold.



3 KU

2. The two boxes below are mathematically similar and both have to be wrapped with decorative paper.

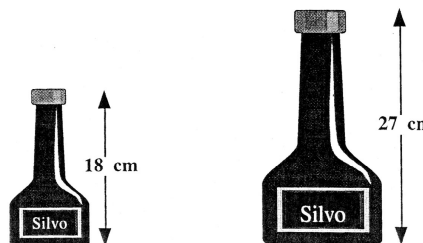


If it requires 3.27 m^2 of paper to cover the large box, calculate the amount of paper needed to cover the smaller box.

3 KU

3. The diagram shows two bottles of Silvo Shampoo.

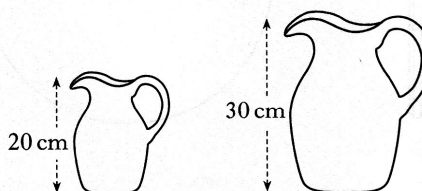
The two bottles are **mathematically similar**, and the cost of the shampoo depends only on the volume of liquid in the bottle.



If the small one costs 80p, what should the large one cost ?

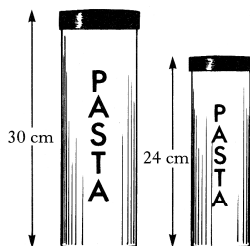
3 KU

4. The diagram shows two jugs which are mathematically similar. The volume of the smaller jug is 0.8 litres. Find the volume of the larger jug.



3 KU

- 5.



The diagram shows two storage jars which are mathematically similar.

The volume of the large jar is 1.2 litres.

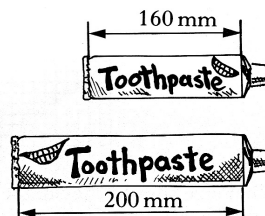
Find the volume of the smaller jar.

Give your answer in litres correct to 2 significant figures.

4 KU

6. The diagram shows two tubes of toothpaste.

Assuming that the tubes are mathematically similar, and that the price of toothpaste depends only on the volume of toothpaste in the tube, what would be the cost of the large tube when the small one costs £1.12?

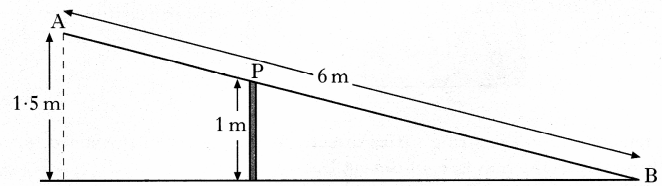


3 KU

Similar Triangles

1. A metal beam, AB, is 6 metres long. It is hinged at the top, P, of a vertical post 1 metre high. When B touches the ground, A is 1.5 metres above the ground, as shown in Figure 1.

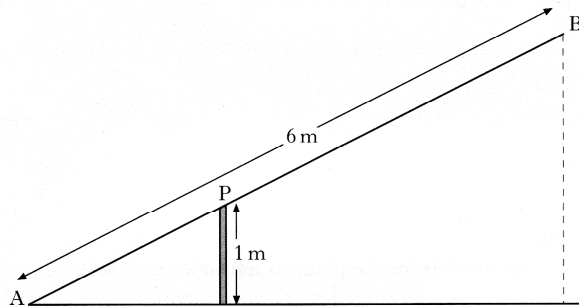
Figure 1



When A comes down to the ground, B rises, as shown in Figure 2.

By calculating the length of AP, or otherwise, find the height of B above the ground.

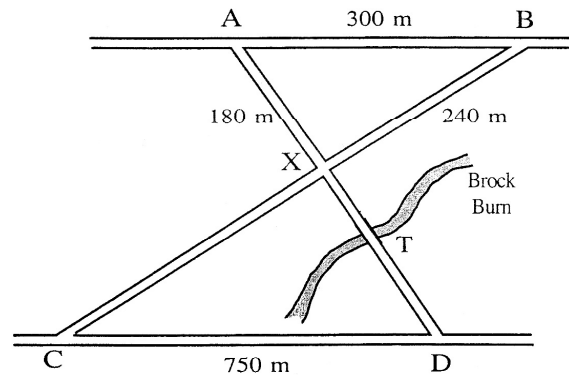
Do not use a scale drawing.



5 RE

2. The road joining A to B is parallel to the road joining C to D in the diagram.

AB = 300 metres,
AX = 180 metres,
BX = 240 metres
and CD = 750 metres.



- a) Prove that the two roads AX and BX are at right angles to one another
- b) The Brock Burn burst its banks at T and the road became impassable. An alternative route had to be found in order to travel from A to D.

3 RE

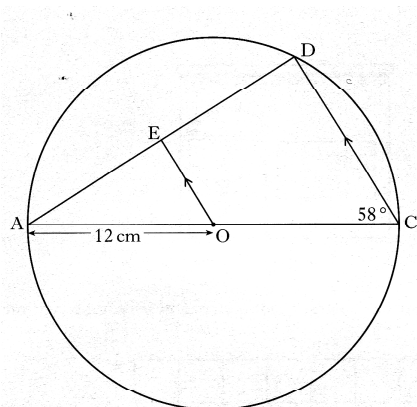
Calculate the length of the shortest route.

3 RE

3. AC is the diameter of the circle, with centre O, and radius 12 centimetres

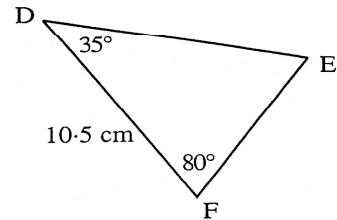
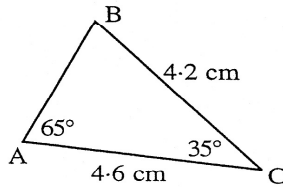
AD is a chord of the circle.
OE is parallel to CD
Angle ACD is 58°

Calculate the length of ED.



4 KU

4. Study the two triangles shown.

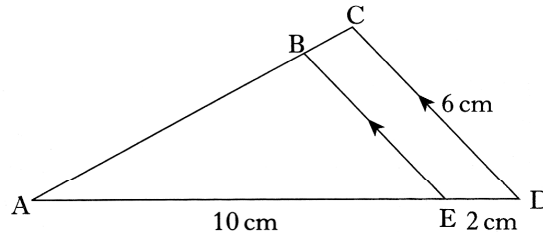


- a) Explain clearly why the two triangles must be similar. 1 KU
 b) Use the fact that the two triangles are similar to calculate the length of the line DE. 2 KU

5. Triangles ABE and ACD with some of their measurements are shown opposite.

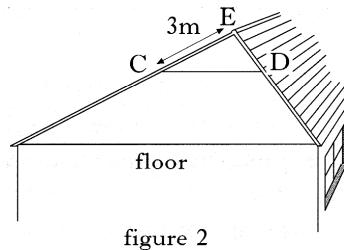
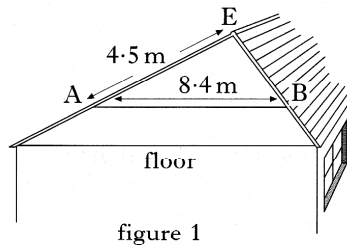
Triangle ABE is similar to triangle ACD.
 Calculate the length of BE.

Do not use a scale drawing.



3 KU

6. The brown family want to convert the roof space in their bungalow into an extra room.



The position, AB, of the wooden beam must be changed to position CD, as shown in figure 2.

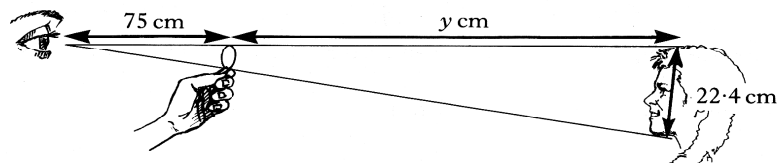
The wooden beam must always be parallel to the floor.

By considering the similar triangles EAB and ECD, calculate the length of the wooden beam in position CD.

Do not use a scale drawing.

3 KU

7. By holding a 10 pence coin at arms' length, it is possible to cover exactly the face of a person standing a distance away.



The diameter of the 10 pence coin is 2.8 cm and the length from the top to the bottom of the person's face is 22.4 cm.

If the distance from the observer's eye to the top of the coin is 75 cm, find the distance from the top of the 10 pence coin to the top of the person's head.

4 KU

6. Pythagoras

NB There is some overlap between these questions and those on the Circle, Similar Triangles and Trigonometry.

1. A sheep shelter is part of a cylinder as shown in Figure 1.

It is 6 metres wide and 2 metres high.

The cross-section of the shelter is a segment of a circle with centre O , as shown in Figure 2.

OB is the radius of the circle.

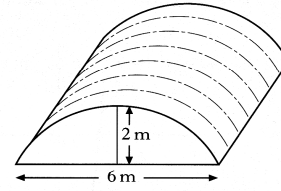


Figure 1

Calculate the length of OB .

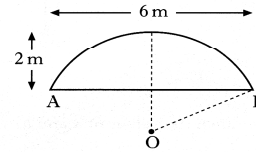


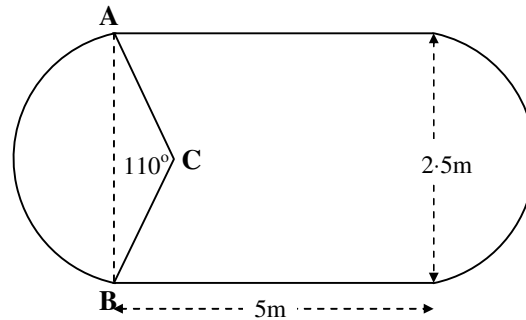
Figure 2

4 RE

2. A large shop display table is in the shape of a rectangle with a circle segment at both ends, as shown in the diagram below.

The rectangle at the centre measures 5 metres by 2.5 metres.

AC and BC are radii of the circle and angle ACB is 110° .



- (a) Show that AC , the radius of the segment, is 1.53 m correct to 3 significant figures.

3 RE

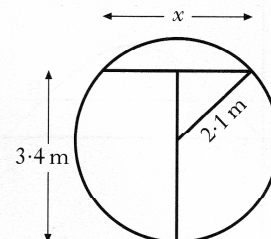
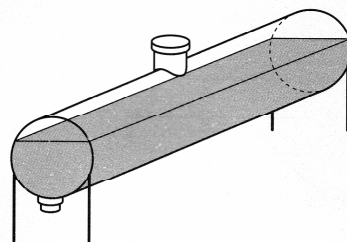
- (b) To stand comfortably around this table it is estimated that an average person requires 75 cm of table edge.

How many people can stand comfortably at the table described above?

4 RE

3. An oil tank has a circular cross section of radius 2.1 metres.

It is filled to a depth of 3.4 metres.



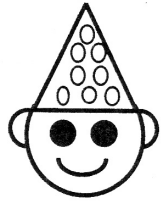
- a) Calculate x , the width in metres of the oil surface.

3 KU

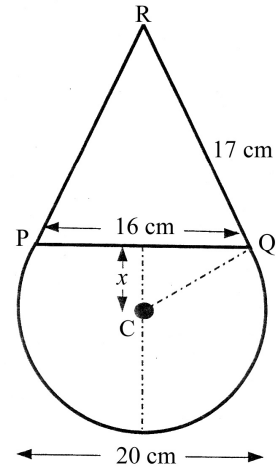
- b) What other depth of oil would give the same surface width.

1 RE

4. A clown's face consists of an isosceles triangle PQR on top of a sector of a circle.



The diameter of the circle is 20 centimetres.
The base of the triangle is 16 centimetres and its sloping sides are 17 centimetres long.



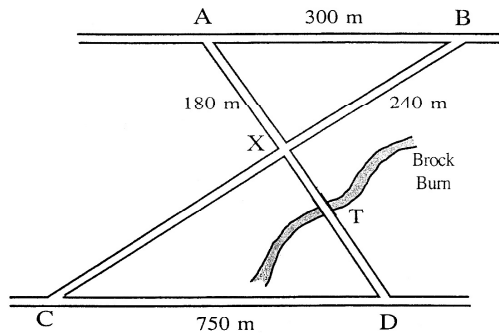
- a) Calculate x , the distance in centimeters from the centre of the circle to the base of the triangle.
b) Calculate the total height of the figure.

3 KU

3 RE

5. The road joining A to B is parallel to the road joining C to D in the diagram.

$AB = 300$ metres,
 $AX = 180$ metres,
 $BX = 240$ metres
and $CD = 750$ metres.



- a) Prove that the two roads AX and BX are at right angles to one another
b) The Brock Burn burst its banks at T and the road became impassable. An alternative route had to be found in order to travel from A to D.

3 RE

Calculate the length of the shortest route.

3 RE

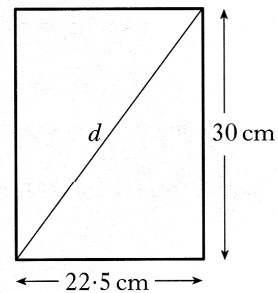
6. A rectangular picture frame is to be made.

It is 30 centimetres high and 22.5 centimetres wide, as shown.

To check that the frame is rectangular, the diagonal, d , is measured.

It is 37.3 centimetres long.

Is the frame rectangular ?



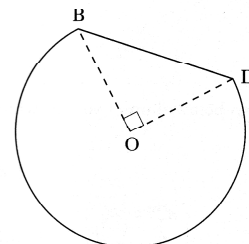
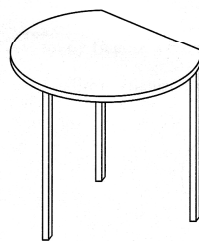
4 RE

7. The diagram shows a table whose top is in the shape of part of a circle with centre, O, and radius 60 centimetres.

BD is a straight line.

Angle BOD is 90° .

Calculate the perimeter of the table top.

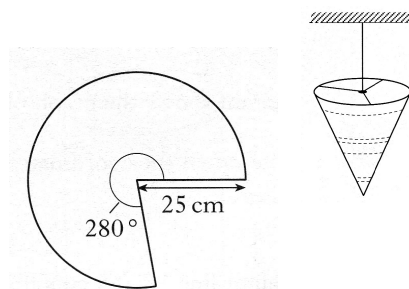


3 RE

8. A lampshade is made in the shape of a cone, as shown.

The shape of the material used for the lampshade is a sector of a circle.

The circle has radius 25 centimetres and the angle of the sector is 280°

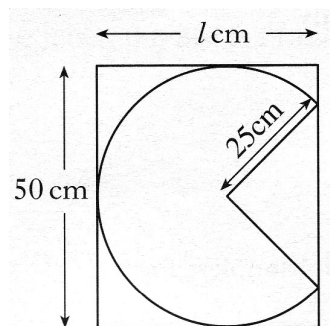


- a) Find the area of the sector of the circle.

3 KU

Each sector is cut from a rectangular piece of material, 50 centimetres wide.

- b) Find to the nearest centimetre the minimum length l , required for the piece of material.

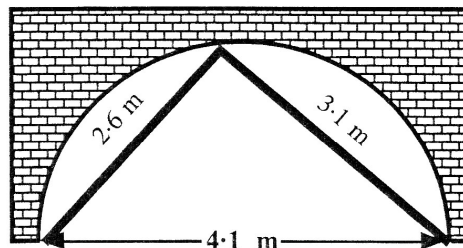


4 RE

9. The central semi-circular archway under a bridge is to be strengthened.

While the work is being carried out, 2 metal beams are to be set in place to support the archway.

For safety reasons, the beams have to just meet on the circumference of the arch.



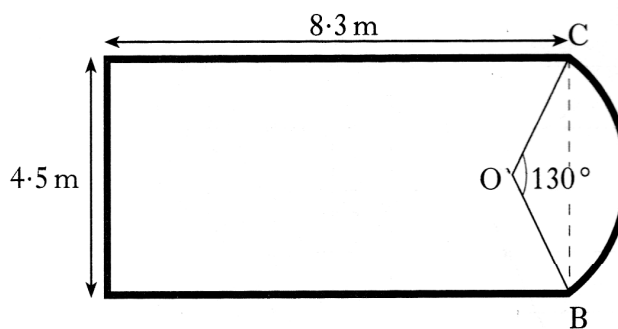
Will the beams fit this archway which is 4.1 metres wide ?

4 RE

10. The diagram shows a ceiling in the shape of a rectangle and a segment of a circle.

The rectangle measures 8.3 metres by 4.5 metres.

OB and OC are radii of the circle and angle BOC is 130° .



- a) Find the length of OB.

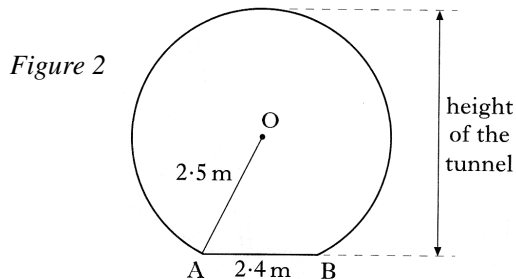
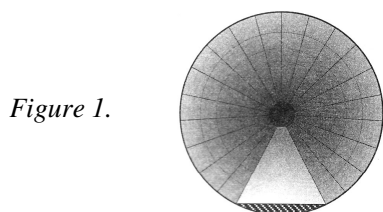
2 RE

A border has to be fitted around the perimeter of the ceiling.

- b) Find the length of border required.

4 RE

11. Figure 1 shows the circular cross section of a tunnel with a horizontal floor.



In figure 2, AB represents the floor.
AB is 2.4 metres.

The radius, OA, of the cross-section is 2.5 metres.

Find the height of the tunnel.

4 KU

6. The diagram shows the design of an earring.

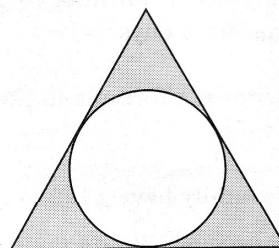
The earring consists of a circle inside an equilateral triangle.

The sides of the triangle are tangents to the circle.

The radius of the circle is 8 mm

The distance from the centre of the circle to **each** vertex of the triangle is 17mm.

Calculate the perimeter of the triangle.



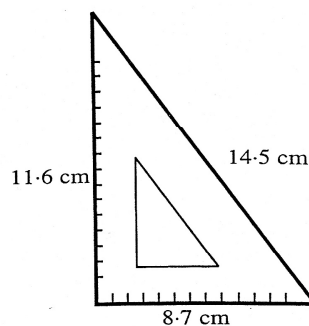
4 RE

10. MATRIX is a company which makes mathematical instruments.

They intend to make a new size of set square which **must have a perfect right angle** at one of its corners..

If the set square has sides of length 8.7 cm, 11.6 cm and 14.5 cm, will it be acceptable.

(Give reasons for your answer).



4 RE

11. Figure 1 shows a road bridge.

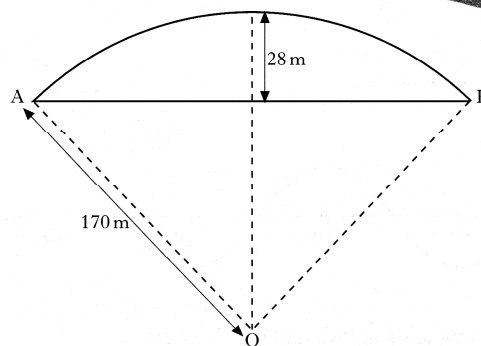
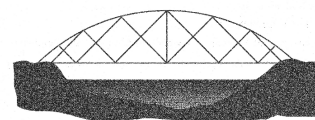
The curved part of the bridge is formed from the arc of a circle, centre O, as shown in figure 2.

OA and OB are radii of length 170 metres.

The height of the middle of the bridge above its ends is 28 metres as shown in figure 2.

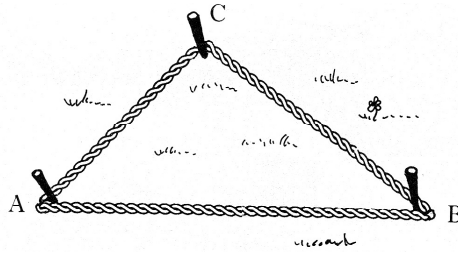
Calculate the horizontal distance, AB.

Do not use a scale drawing.



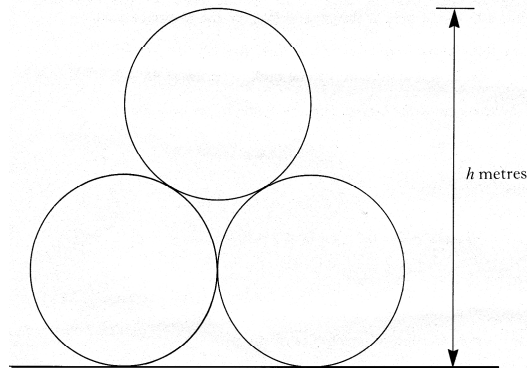
4 KU

12. A loop of rope is used to mark out a triangular plot, ABC.
 The loop of rope measures 6 metres.
 Pegs are positioned at A and B such that AB is 2.5 metres.
 The third peg is positioned at C such that BC is 2 metres.
 Prove that angle ACB = 90°.
Do not use a scale drawing.



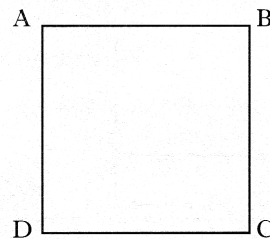
4 KU

13. Three pipes are stored on horizontal ground as shown in the diagram.
 Each pipe has a circular cross-section with radius 1 metre.
 Calculate the height, h metres, of the stacked pipes. (Ignore the thickness of the pipes.)
Give your answer in metres correct, to two decimal places.



4 RE

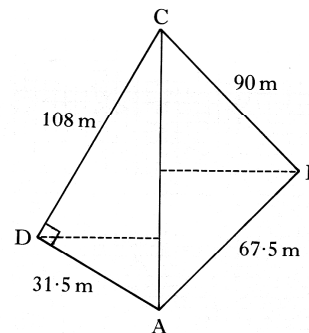
12. a) ABCD is a square of side 2 cms
 Write down the ratio of the length AB to the length of AC.
 b) Show that in every square, the ratio of the length of a side to the length of a diagonal is $1 : \sqrt{2}$



2 KU

3 RE

13. A school's playing fields have recently been surveyed and the following plan produced.
 The plan is not drawn to scale.
 AB = 67.5 metres
 BC = 90 metres
 AD = 31.5 metres
 DC = 108 metres
 Angle ADC = 90°
 Without doing any further measurements, the surveyor realises that angle ABC is a right angle.
 Prove that angle ABC = 90°



4 KU

This next question is quite an involved one from 1990. It is unlikely to be set today, however, if you can do this then you have demonstrated an excellent understanding of mathematics.

14. The diagram of a rivet is shown opposite.
The body of the rivet is in the shape of a cylinder.

The head of the rivet is a cap of a sphere of radius R , which is obtained as shown in figure 2.



Figure 1

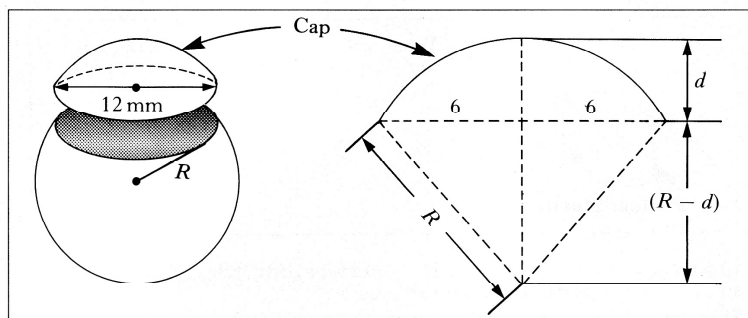


Figure 2

- a) Find the value of R for this cap of width 12mm when its height in mm is given by $d = 2.4 R$
- b) The length of the cylindrical body of this rivet is 8mm and the diameter of the base is 5mm.

Assuming that the volume of the cap of the sphere is given by

$$V = \frac{1}{3}\pi d^2 (3R - d)$$

show that the total volume of the rivet

is $\frac{262\pi}{3} \text{ mm}^3$

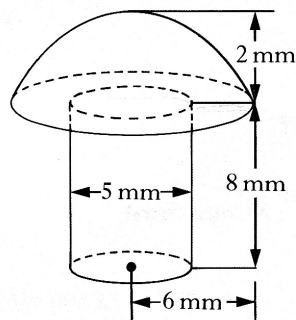


Figure 3

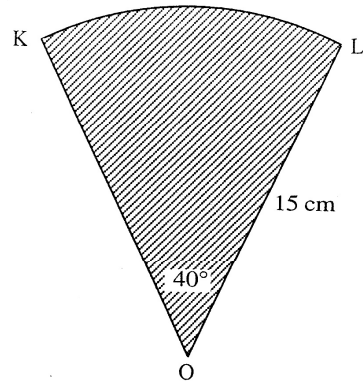
4 RE

7. The Circle

NB There is considerable overlap between these questions and those on Pythagoras and Trigonometry.

1. Sector KOL of a circle centre O and radius 15 centimetres is shown opposite.

Calculate the area of this sector.

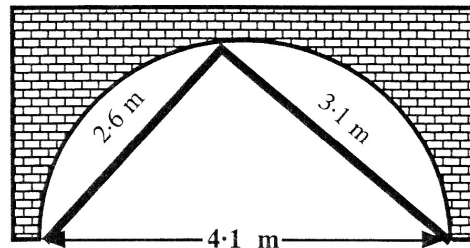


2 KU

2. The central semi-circular archway under a bridge is to be strengthened.

While the work is being carried out, 2 metal beams are to be set in place to support the archway.

For safety reasons, the beams have to just meet on the circumference of the arch.

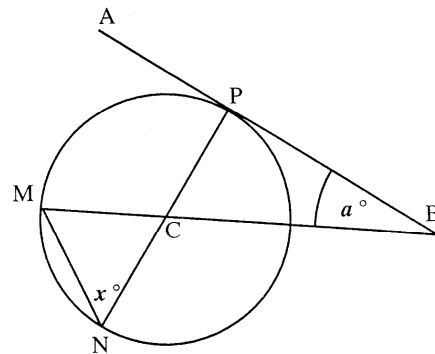


Will the beams fit this archway which is 4.1 metres wide ?


4 RE

3. AB is a tangent to the circle with **centre C**. It meets the circle at the point P.

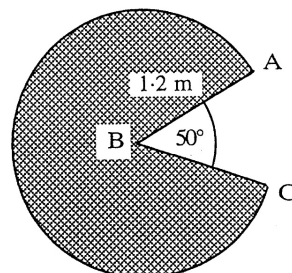
Use the information in the diagram to find an expression for x in terms of a .



3 RE

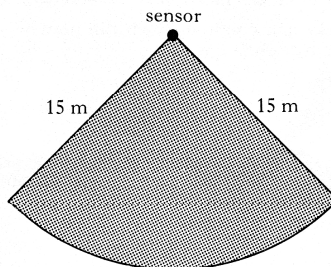
4.  June is replacing the fabric on her garden parasol. She uses a sector of a circle, with radius 1.2 metres.

Calculate the area of fabric needed to replace the old material.



4 KU

5. A sensor in a security system covers a horizontal area in the shape of a sector of a circle of radius 15 m.



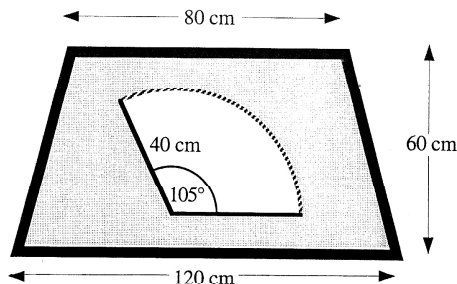
The area of the sector is 200 square metres.
Find the length of the arc of the sector.

4 RE

6. The diagram shows the rear wiper on a car's back window.

The rear glass is in the shape of a **trapezium** with sizes given.

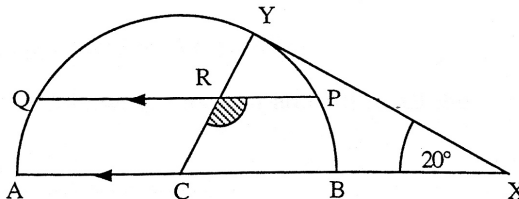
The wiper blade is 40 centimetres long and it sweeps through an angle of 105° .



Calculate the area of glass **NOT** cleaned by the wiper blade.

4 RE

7. In this diagram, AB is the diameter of the circle, centre C.
X is a point of the line AB extended.
XY is a tangent from X.
QP is parallel to AB.



If $\angle YXC = 20^\circ$, calculate the size of the shaded angle ($\angle PRC$)

3 KU

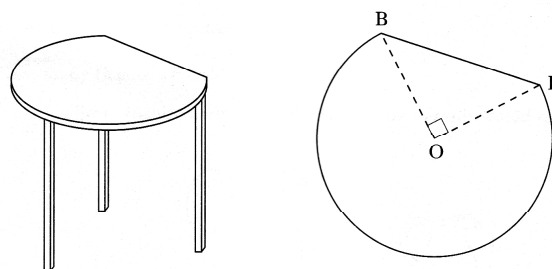
(explain how you produced your answer)

8. The diagram shows a table whose top is in the shape of part of a circle with centre, O, and radius 60 centimetres.

BD is a straight line.

Angle BOD is 90° .

Calculate the perimeter of the table top.



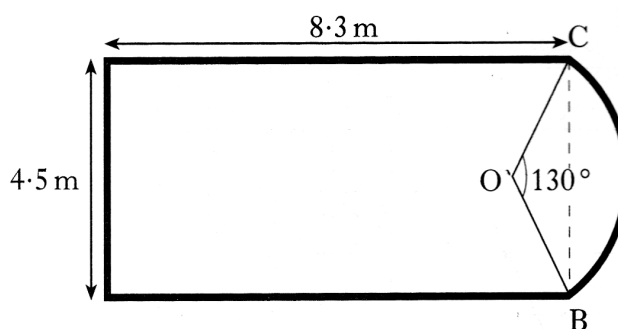
3 RE

9. The diagram shows a ceiling in the shape of a rectangle and a segment of a circle.

The rectangle measures 8.3 metres by 4.5 metres.

OB and OC are radii of the circle and angle BOC is 130° .

- a) Find the length of OB.



2 RE

A border has to be fitted around the perimeter of the ceiling.

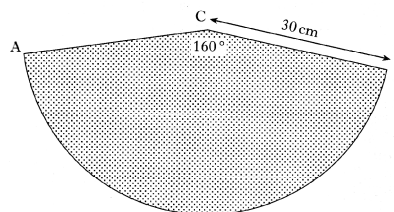
- b) Find the length of border required.

4 RE

10. The diagram shows a sector of a circle, centre, C.

Angle ACB is 160° ,
and the radius of the circle is 30 cm.

Calculate the length of the arc AB.



3 KU

11. The diagram shows the design of an earring.

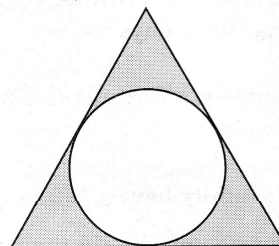
The earring consists of a circle
inside an equilateral triangle.

The sides of the triangle are tangents to the circle.

The radius of the circle is 8 mm

The distance from the centre of the circle
to **each** vertex of the triangle is 17mm.

Calculate the perimeter of the triangle.



4 RE

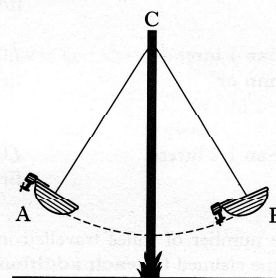
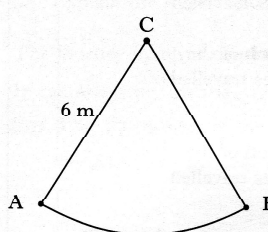
12. The boat on a carnival ride travels along an arc of a circle, centre C.

The boat is attached to C
by a rod 6 metres long.

The rod swings from position CA
to position CB.

The length of the arc AB is 7 metres.

Find the angle through which the
rod swings from position A to position B.



4 RE

13. The diagram shows a tent.

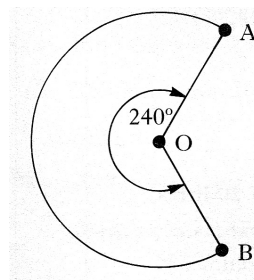
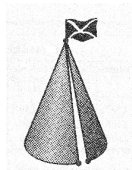
The shape of the material used
to make the tent is a sector of a
circle as shown in the diagram.

O is the centre of the circle.

OA and OB are radii of length 3 metres.

Angle AOB is 240°

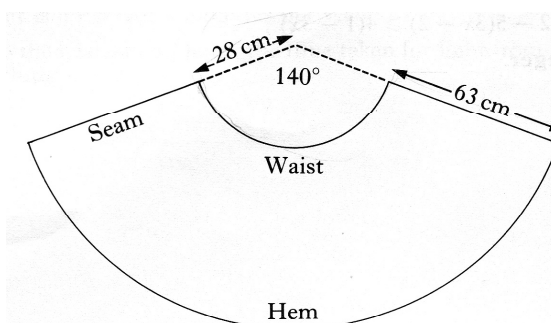
Calculate the area of this piece of material.



3 KU

14. The pattern for a skirt consists
of part of the sector of a circle.

Calculate the length of the waist
shown on the pattern.

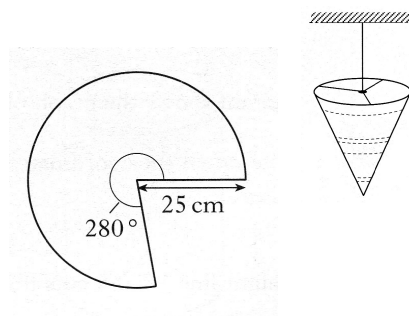


3 KU

15. A lampshade is made in the shape of a cone, as shown.

The shape of the material used for the lampshade is a sector of a circle.

The circle has radius 25 centimetres and the angle of the sector is 280°

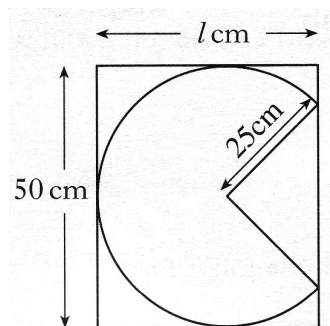


- a) Find the area of the sector of the circle.

3 KU

Each sector is cut from a rectangular piece of material, 50 centimetres wide.

- b) Find to the nearest centimetre the minimum length l , required for the piece of material.

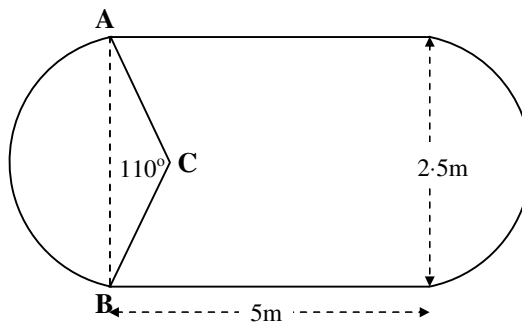


4 RE

16. A large shop display table is in the shape of a rectangle with a circle segment at both ends, as shown in the diagram below.

The rectangle at the centre measures 5 metres by 2.5 metres.

AC and BC are radii of the circle and angle ACB is 110° .



- (a) Show that AC, the radius of the segment, is 1.53 m correct to 3 significant figures.
- (b) To stand comfortably around this table it is estimated that an average person requires 75 cm of table edge.

3 RE

How many people can stand comfortably at the table described above?

4 RE