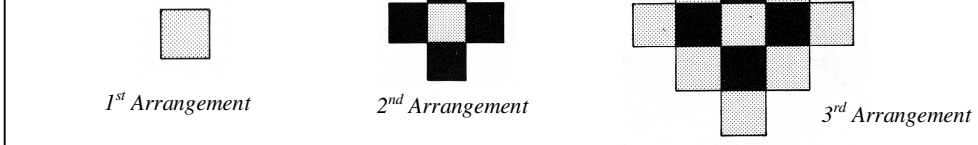
12. A large floor is to be covered with black and grey square tiles to make a chequered pattern.

The person laying the tiles must start at the centre of the floor and work outwards.

The instructions are as follows.



1st Arrangement *2nd Arrangement* *3rd Arrangement*

1. Lay a grey tile in the centre of the floor

2. Place black tiles against the edges of the grey tiles

3. Place grey tiles against the edges of all the black tiles

4. Place black tiles against the edges of all the grey tiles.

5. And so on …..

a) How many tiles are there in the 4th arrangement? 2 RE

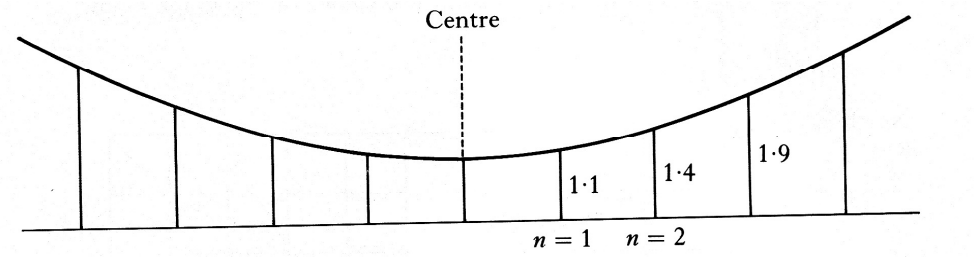
b) The number of tiles, *T*, needed to make the *N* th arrangement is given by the formula

$$T = 2N^2 + aN + b$$

Find the values of *a* and *b*. 4 RE

13. The heights in metres of the vertical rods of an early suspension bridge, as you move out from the centre, form the sequence

1.1, 1.4, 1.9, 2.6, ……



a) What are the likely heights of the 5th and 6th rods in this sequence 2 RE

b) The height, h metres, of the n^{th} rod in the sequence is given by the formula

$$h = A + bn^2$$

Find the values of A and b and write down the formula. 4 RE

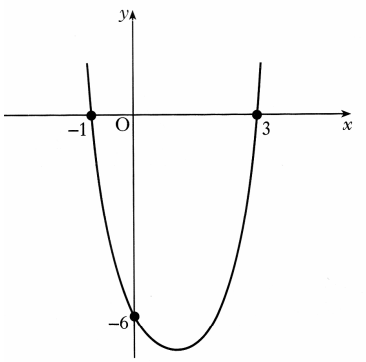
12. Functions

Properties of the parabola

1. The diagram shows part of the graph of a quadratic function, with equation of the form

$$y = k(x-a)(x-b)$$

The graph cuts the y-axis at (0, -6) and the x-axis at (-1, 0) and (3, 0)

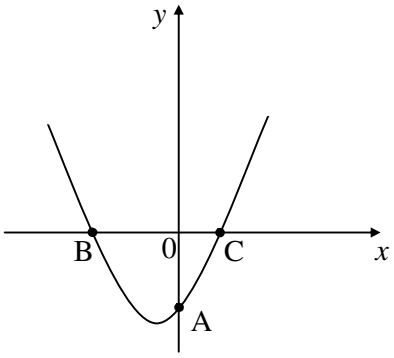


a) Write down the values of a and b. 2 KU

b) Calculate the value of k. 2 KU

c) Find the coordinates of the minimum turning point of the function 2 RE

2. The graph shown has equation $y = x^2 + x - 12$.



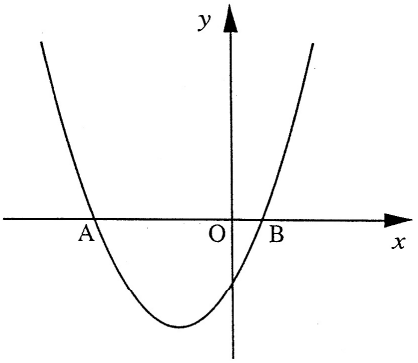
(a) Find the coordinates of A, the point where the curve cuts the y – axis. 1 RE

(b) Find the coordinates of B and C, the points where the curve cuts the x – axis. 3 RE

(c) Find the coordinates of the minimum turning point. 2 RE

3. The graph shows the parabola

$$y = 3x^2 + 7x - 2$$



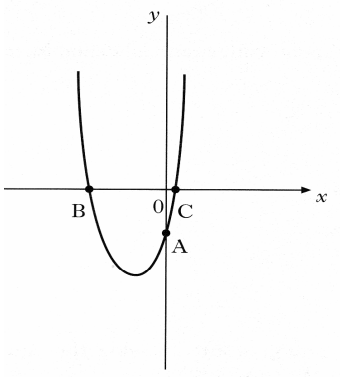
By solving the quadratic equation

$$3x^2 + 7x - 2 = 0$$

find the coordinates of point A.

Give your answer **correct to 2 decimal places**. 4 KU

4. The diagram below shows part of the graph of $y = 4x^2 + 4x - 3$



The graph cuts the y-axis at A and the x-axis at B and C.

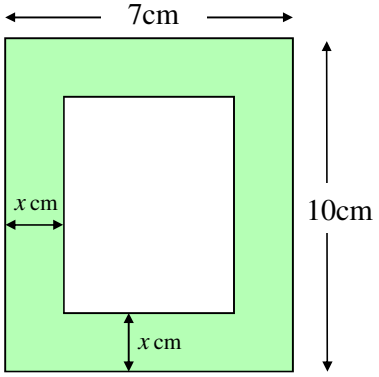
a) Write down the coordinates of A 1 KU

b) Find the co-ordinates of B and C. 3 KU

c) Calculate the minimum value of $4x^2 + 4x - 3$ 2 RE

Applications of the parabola

1. Jane found a small photo-frame and decided to put one of her favourite photographs in it. The diagram below shows the dimensions of the frame.



The width of the wooden surround is x cm.

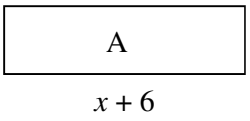
Unfortunately the glass in the centre of the frame was cracked and had to be replaced.

(a) Show that the area of glass needed for the centre of the frame can be given by the formula

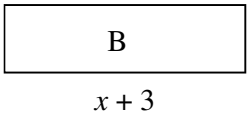
$$A = \left(4x^2 - 34x + 70 \right) \text{ cm}^2$$ 4 RE

(b) If the area of glass needed was 28cm^2, find a possible value for x. 3 KU

2. Rectangle A, shown opposite, has length $x + 6$ units and breadth $x - 1$ units.



Rectangle B has length $x + 3$ units and breadth 3 units.



a) Write down expressions, in terms of x, for the area of Rectangle A and the area of Rectangle B. 2 KU

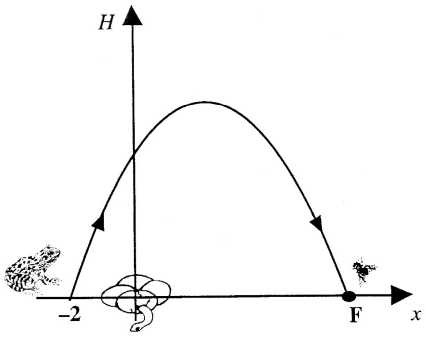
b) Given that both rectangles have the same area for a particular value of x, form an equation using your answers to part (a) and solve it to find this value of x. 3 RE

3. A frog is sitting 2 feet to the left of a snake.

The frog then notices a fly sitting on a rock on the other side of the snake. As the frog leaps over the snake to catch the fly, its path is described by the parabola with equation

$$H = 8 + 2x - x^2$$

where H is the height of the frog above the ground.



a) By considering the quadratic equation:

$$8 + 2x - x^2 = 0$$

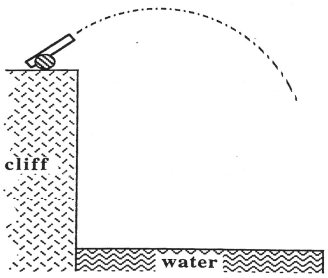
find the co-ordinates of the point F, where the fly is sitting, and hence write down how far away the fly is from the frog. 4 KU

b) How high above the ground does the frog reach on its jump ? 3 RE

4. When a shell is fired from a cannon on top of a cliff, the height of the shell above the water surface is given by the formula:

$$H(t) = 9 + 6t - 3t^2$$

where t is the time in seconds and $H(t)$ is the height in metres after t seconds.



Calculate the height of the shell after 3 seconds.

Explain what your answer indicates. 2 KU

5. A gardener creates an L-shaped flower bed. He uses the house walls and concrete edging for the boundary as shown in figure 1.

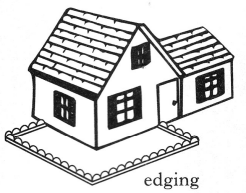


figure 1.

He plans his flower bed as shown in figure 2.

a) He uses a total of **6 meters of edging.**

AB = ED = x metres.
BC = DC

Show that the length in metres, of BC, can be expressed as BC = 3 – x. 2 RE

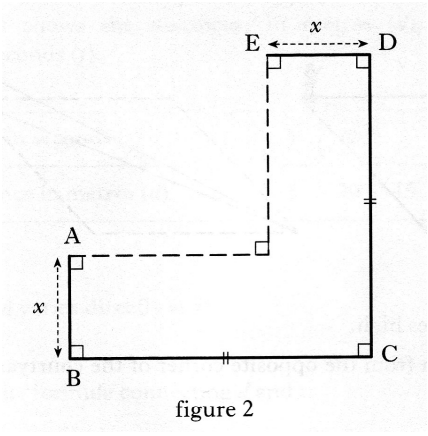


figure 2

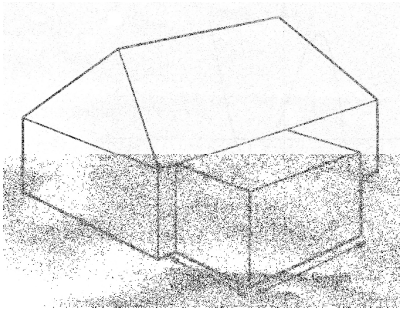
b) Hence show that the area, A, in square metres, of the flower bed can be expressed as

$$A = 6x - 3x^2$$ 3 RE

c) Calculate **algebraically** the maximum area of the flower bed. 3 RE

6. A family want to build an extension at the rear of their house.

An architect advises that the extension should have its length 2 metres more than its width.



a) If the width of the extension is w metres, write down an expression for its length. 1 KU

Planning regulations state that the area of the ground floor of the extension must not exceed 40% of the area of the ground floor of the original house.

b) The ground floor of the original house is 12 metres by 10 metres.
Show that, if the largest extension is to be built, $w^2 + 2w - 48 = 0$. 3 RE

c) Find the dimensions of the largest extension which can be built. 2 RE

7. A rectangular sheet of plastic 18 cm by 100 cm is used to make a gutter for draining rain water.

The gutter is made by bending the sheet of plastic as shown below in diagram 1.

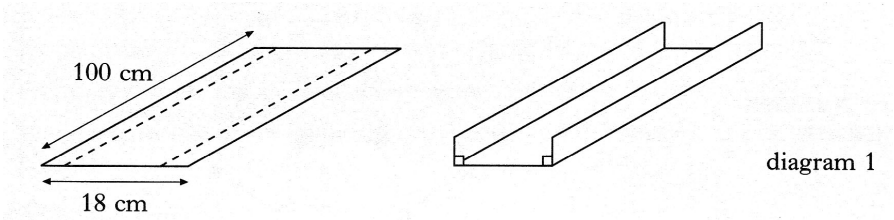


diagram 1

a) The depth of the gutter is x centimetres as shown in diagram 2 below.

Write down an expression in x for the width of the gutter. 1 KU

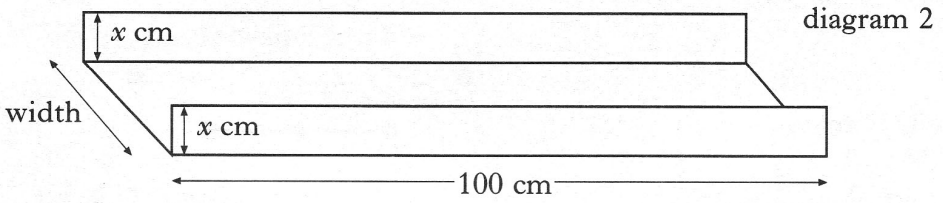


diagram 2

b) Show that the volume, V cubic centimetres, of this gutter is given by

$$V = 1800x - 200x^2$$ 2 RE

c) Find the dimensions of the gutter which has the largest volume.

Show clearly all your working. 4 RE

13. Making & Using Formulae

1. A rectangular clipboard has a triangular plastic pocket attached as shown in Figure 1.

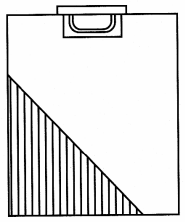


Figure 1

The pocket is attached along edges TD and DB as shown in Figure 2.
B is x centimeters from the corner C.

The length of the clipboard is $4x$ centimeters and the breadth is $3x$ centimeters.

The area of the pocket is a quarter of the area of the clipboard.

Find in terms of x, the length of TD. 4 RE

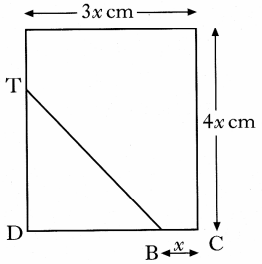


Figure 2

2. The number of diagonals, d, in a polygon with n sides is given by the formula:

$$d = \frac{n(n-3)}{2}$$

A polygon has 20 diagonals.

How many sides does it have? 4 RE

3. Esther has a new mobile phone and considers the following daily rates.

Easy Call	**Green Call**
25 pence per minute for the first 3 minutes 5 pence per minute **after** the first three minutes.	40 pence per minute for the first 2 minutes 2 pence per minute **after** the first two minutes.

a) For Easy Call, find the cost of ten minutes in a day. 1 KU

b) For Easy Call, find a formula for the cost of "m" minutes in a day, $m > 3$ 1 RE

c) For Green Call, find a formula for the cost of "m" minutes in a day, $m > 2$ 1 RE

d) Green Call claims that its system is cheaper.

Find **algebraically**, the least number of minutes (to the nearest minute) which must be used each day for this claim to be true. 3 RE

4. The intensity of light, I, emerging after passing through a liquid with concentration, c, is given by the equation

$$I = \frac{20}{2^c} \quad c \geq 0$$

a) Find the intensity of light when the concentration is 3. 1 KU

b) Find the concentration of the liquid when the intensity is 10 2 KU

c) What is the maximum possible intensity ? 3 RE

5. A rectangular wall vent is 30 centimetres long and 20 centimetres wide.

It is to be enlarged by increasing **both** the length and the width by x centimetres.

a) Write down the length of the new vent. 1 RE

b) Show that the Area, A, square centimeters, of the new vent is given by

$$A = x^2 + 50x + 600$$ 2 RE

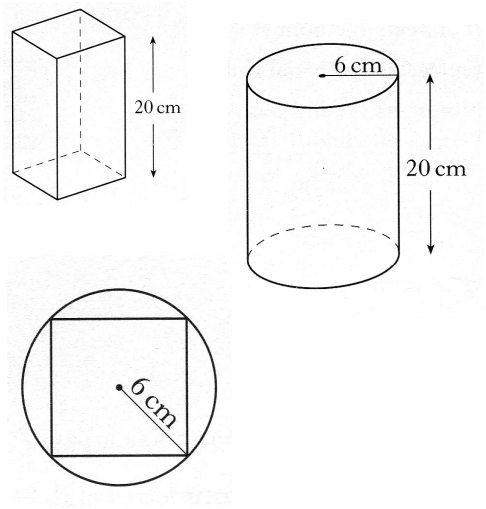
c) The area of the new vent **must** be **at least** 40% more than the original area.

Find the minimum dimensions to the nearest centimeters, of the new vent. 5 RE

6. A glass vase, in the shape of a cuboid with a square base is 20 centimetres high.

It is packed in a cardboard cylinder with radius 6 centimetres and height 20 centimetres.

The corners of the vase touch the Inside of the cylinder as shown.



Show that the volume of the space between the vase and the cylinder is $720(\pi - 2)$ cubic centimetres. 5 RE

7. The cost of renting one of three apartments in Greece depends on the number of people sharing.

If there are **less** than the standard number of people sharing an apartment, (known as under-occupancy), an extra fee is charged.

If there are **more** than the standard number, then a reduction is given to **every** person in the room, *(based on **each** extra adult).*

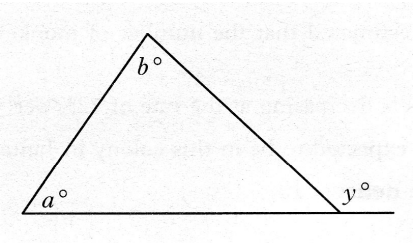
The table below shows how the cost is calculated.

Style of Appartment	*No. Rooms*	*Cost per person per week*	*Based on number sharing*	*Under-occupancy extra fee per person (£)*	*Reduction per extra adult (£)*
Mailia	1	425	2	40	30 (max. 2 extra)
Mavrikos	2	310	4	45	25 (max. 2 extra)
Tsilivi	3	450	6	55	40 (max. 4 extra)

a) Find the total cost of 4 adults staying at Malia Appartments for 1 week. 2 RE

b) Find a formula to calculate the total cost £C, of P people staying at Malia for 1 week, where P is greater than 2 but less than 5. 3 RE

8. Use the information in the diagram to find a relationship connecting a, b and y



2 RE

9. Anna hired a mobile phone at a fixed charge of £17.50 per month.

She is also charged for her total call time each month.

15 minutes of this total call time are **free**. The rest of her call time is charged at 35 pence per minute.

a) What is the total cost for Anna's phone in a month when her **total call time** is 42 minutes. 2 KU

b) Write down a formula for the total cost, £C, for Anna's phone in a month when her total call time is t minutes, where $t \geq 15$. 3 RE

10. A gardener creates an L-shaped flower bed. He uses the house walls and concrete edging for the boundary as shown in figure 1.

figure 1.

He plans his flower bed as shown in figure 2.

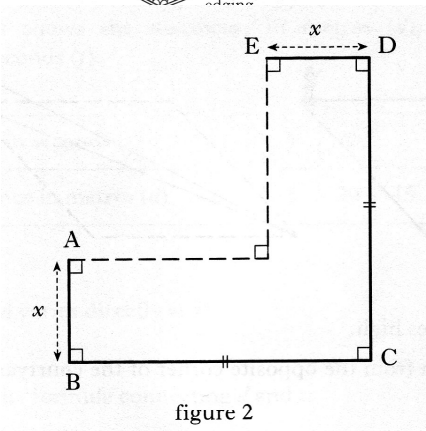


figure 2

a) He uses a total of **6 meters of edging.**

AB = ED = x metres.
BC = DC

Show that the length in metres, of BC, can be expressed as BC = $3 - x$. 2 RE

b) Hence show that the area, A, in square metres, of the flower bed can be expressed as

$$A = 6x - 3x^2$$

3 RE

c) Calculate **algebraically** the maximum area of the flower bed. 3 RE

11. The cost of taking a school group to the theatre can be calculated from the information shown below.

*** 1 adult goes free for every 10 pupils ***

Number of pupils	*Cost per pupil*	*Cost per paying adult*
Less than 10	£5.00	£8.00
10 to 19	£4.50	£7.00
20 to 29	£4.00	£6.00
30 to 39	£3.00	£5.00

a) Find the cost for a group of 12 pupils and 3 adults. 2 RE

b) Write down a formula to find the cost, £C, of taking a group of p pupils and d adults where $20 \leq p \leq 29$. 4 RE

12. Traffic authorities are investigating the number of cars travelling along a busy stretch of road.

They assume that all cars are travelling at a speed of v metres per second.

The number of cars, N, which pass a particular point on the road in one minute is given by the formula

$$N = \frac{30v}{2+v}$$

In one minute, 26 cars pass a point on the road.

Find the speed of the cars in metres per second. 3 RE

13. While on holiday, John's family decide to hire a car.

There are two different schemes for hiring the same type of car, Eurocar and Apex.

EUROCAR HIRE	**APEX HIRE**
No deposit required £15 per day	£50 deposit required **plus** £10 per day

a) Write down a formula to find the cost, £C, of hiring the car from Eurocar for d days. 1 KU

b) Write down a formula to find the cost, £C, of hiring the car from Apex for d days. 2 KU

c) John's family have £170 to spend on car hire.

Which scheme should they use to have the car as long as possible?

Show clearly all your working. 4 RE

14. The area, A, of a quadrilateral drawn inside a circle can be found using the formula

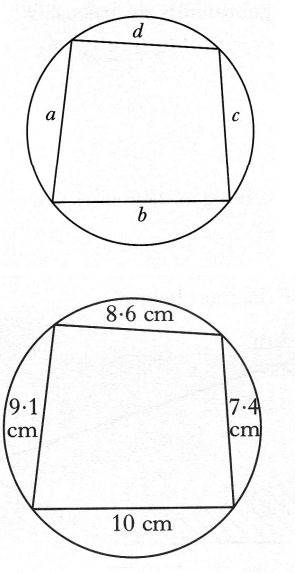
$$A = \sqrt{(s-a)(s-b)(s-c)(s-d)}$$

where

$$s = \frac{(a+b+c+d)}{2}$$

Use this formula to find the area of the quadrilateral shown in the diagram.

Give your answer correct to 2 significant figures. 3 KU



15. The travelling expenses claimed by a salesperson depend on the engine capacity of the car and the number of miles travelled per week as shown in the table below.

ENGINE CAPACITY	EXPENSES PER MILE
Less than or equal to 1 litre	£0.25 for **each** of the first 250 miles travelled
greater than 1 litre but **less than or equal to** 1.2 litres	£0.27 for **each** of the first 250 miles travelled
greater than 1.2 litres	£0.29 for **each** of the first 250 miles travelled
Where the number of miles traveled in a week **is greater than 250**, £0.15 can be claimed for **each additional** mile.	

a) Find the expenses claimed by a salesperson in a week when 550 miles are travelled and the engine capacity is 1.6 litres. 2 RE

b) Write down a formula to find the expenses £E, claimed for t miles travelled, where t is greater than 250, and the engine capacity is 1.6 litres 3 RE

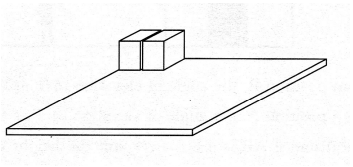
16. The integral part of a positive real number is the part of the number which is an integer.

EXAMPLES **The integral part of 5.6 is 5**
This can be written as **[5.6] = 5**

The integral part of 6.2 is 6
This can be written as **[6.2] = 6**

a) Find [16.7] 1 RE

b) Identical boxes are packed on a board for storage. The boxes are all packed the same way round (two boxes are shown in the diagram).

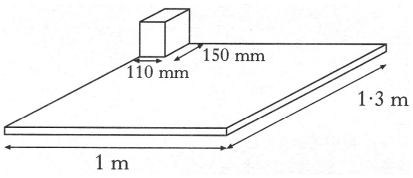


i) The base of each box measures 150 millimetres by 110 millimetres.

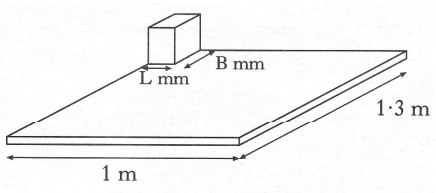
The board measures 1.3 metres by 1 metre.

The number of boxes that can fit along the 1.3 metre length is given by

$\left[\frac{1300}{150}\right]$ Find $\left[\frac{1300}{150}\right]$ 1 RE



ii) Write down an expression for the number of boxes which can be packed on the board shown on the right. 2 RE



17. A rectangular sheet of plastic 18 cm by 100 cm is used to make a gutter for draining rain water.

The gutter is made by bending the sheet of plastic as shown below in diagram 1.

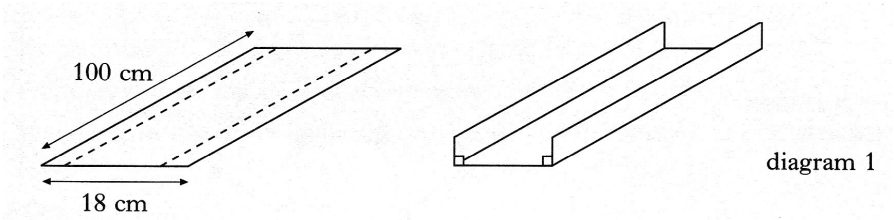


diagram 1

a) The depth of the gutter is x centimetres as shown in diagram 2 below.

Write down an expression in x for the width of the gutter. 1 KU

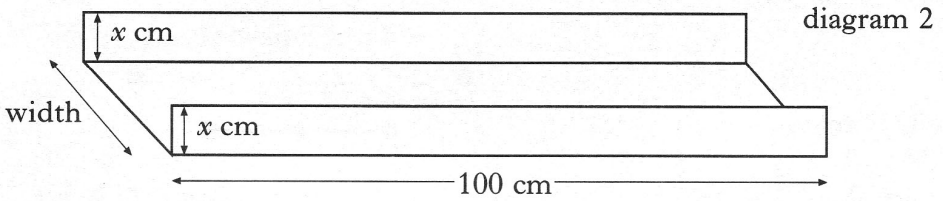


diagram 2

b) Show that the volume, V cubic centimetres, of this gutter is given by

$$V = 1800x - 200x^2$$ 2 RE

c) Find the dimensions of the gutter which has the largest volume.

Show clearly all your working. 4 RE

18. The cost of sending a parcel depends on the weight of the parcel and the time of delivery. The cost is calculated as shown below.

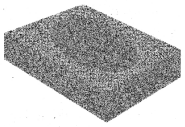
TIME OF DELIVERY	**COST**
by 10 am the next working day	**£18.20** for 10kg and £0.85 for **each extra kg.**
by noon the next working day	**£13.50** for 10kg and £0.75 for **each extra kg.**
by 5 pm the next working day	**£10.50** for 10kg and £0.50 for **each extra kg.**

a) Find the cost of sending a parcel, of weight 14 kg, for delivery **by noon** the next working day. 2 RE

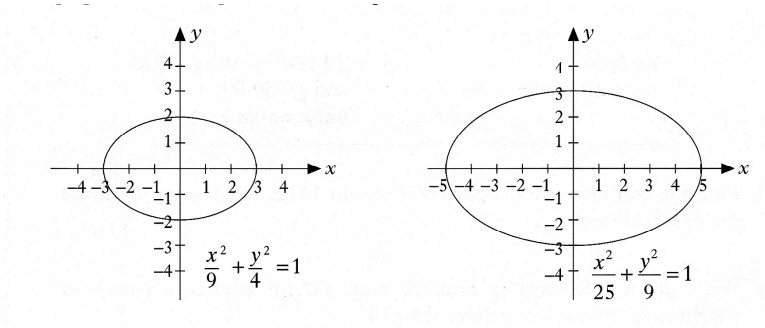
b) Write down a formula to find the cost, £C, of sending a parcel, of weight w kg, where w is greater than 10.

The parcel has to be delivered **by noon** the next working day. 3 RE

19. The opening on this box of tissues is in the shape of an ellipse.

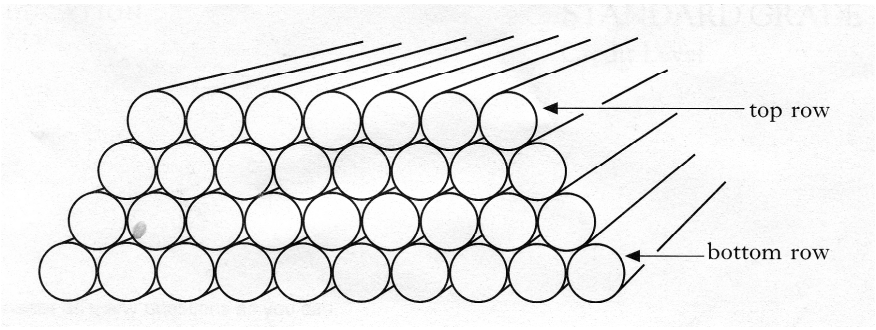


The graphs of two ellipses and their equations are shown below.



Sketch the ellipse with equation $\frac{x^2}{36}+\frac{y^2}{16}=1$ 3 RE

20. Pipes with equal diameters are arranged in a stack.



To find the number of pipes, P, in the stack, the following formula can be used.

$$P=\frac{(b+a)(b-a+1)}{2}$$

where b is the number of pipes on the bottom row and a is the number of pipes on the top row.

a) Use this formula to find the number of pipes in a stack where $b = 40$ and $a = 15$. 1 KU

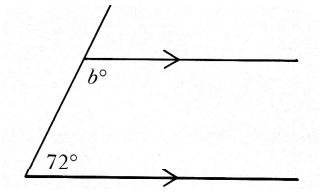
b) In a particular stack, the number of pipes on the bottom row is twice the number on the top row.

Show that in this stack $P=\frac{3a^2+3a}{2}$ where a is the number of pipes on the top row. 3 RE

c) Would it be possible to arrange exactly 975 pipes in the kind of stack described in part b)

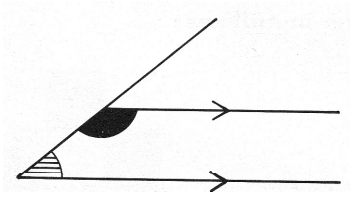
Justify your answer. 3 RE

21. The diagram opposite shows two parallel lines meeting a third at 72°.



a) Find the value of b. 1 KU

b) The diagram opposite shows the general case of two parallel lines meeting a third line.



Prove that in every case, the sum of the shaded angles is 180°. 3 RE

22. An extract from a camping holiday brochure is shown below.

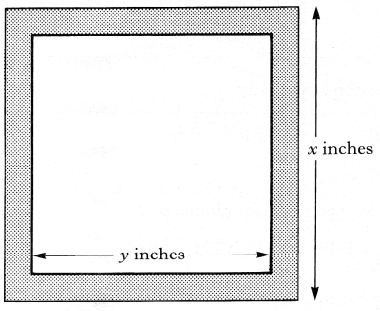
Season	**For 14 nights**					**Over 14 nights**
	Two adults	Each extra adult	Each young adult aged 14 to17	Each child aged 10 to 13	Each child aged 0 to 9	Each additional night per family
Low	£399	£74	£40	Free	Free	£19
Mid	£555	£85	£50	Free	Free	£29
High	£699	£95	£60	£46	Free	£39

a) Find the cost of a holiday for 2 adults and a child, aged 8, for 17 nights during mid-season. 2 RE

b) Write down a formula to find the cost, £C, of a holiday in mid-season for 2 adults and a child aged 8 lasting t nights, where t is greater than 14. 3 RE

23. A square picture frame is shown.

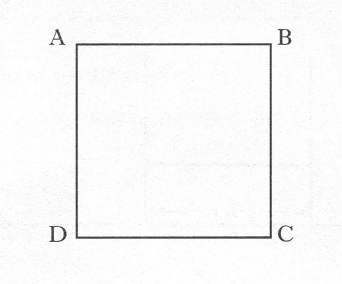
The border of the frame (shaded in the diagram) has uniform width and an area of 48 square inches.



a) Show that $(x-y)(x+y)=48$ 2 RE

b) Given that x and y are whole numbers each greater than 10, find suitable replacements for x and y. 3 RE

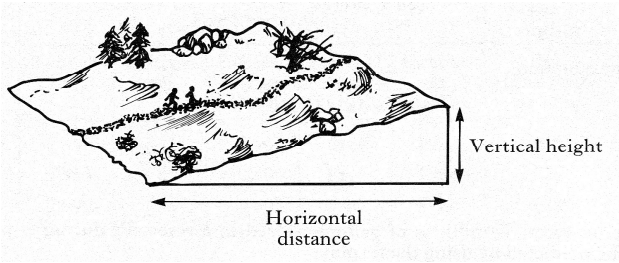
24. a) ABCD is a square of side 2 cms



Write down the ratio of the length AB to the length of AC. 2 KU

b) Show that in every square, the ratio of the length of a side to the length of a diagonal is $1 : \sqrt{2}$ 3 RE

25. The total time a walk takes in hillwalking depends on the horizontal distance covered (h kilometres) and the vertical height climbed v metres.



For **each kilometre** of horizontal distance, 12 minutes should be allowed.

a) i) Write down the time which should be allowed for h kilometres of horizontal distance. 1 KU

ii) for **each 100 metres** of vertical height, 10 minutes should be allowed.

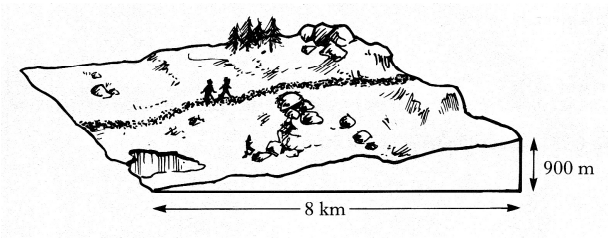
Write down the time which should be allowed for v metres of vertical height. 2 RE

iii) Show that the **total** time T **hours** which should be allowed for the walk is given by the formula

$$T = \frac{120h + v}{600}$$ 3 RE

b) For safety reasons, hillwalkers should be off the hills by 1900 hours.

Would it be safe to start the walk shown at 1300 hours?



3

27. Mr and Mrs Paton want to have their house valued before putting it up for sale.

The fee they have to pay for having this done depends on the value of their house.
The fee is calculated as follows

Value of house	Fee to be paid
First £2000 of value	£5.00
Each additional £500 up to £15000	£1.00 per £500
Each additional £1000 over £15000	£1.00 per £1000

a) The Paton's house is valued at £33 000
What fee will they have to pay? 4 RE

b) Write down a formula to find the total fee payable when a house is valued at £P thousand, where P is a whole number greater than 15. 3 RE

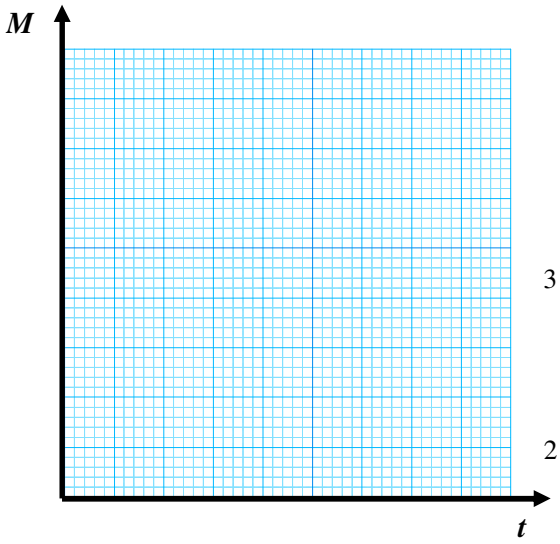
26. The mass, M grams, of a given radio-active isotope decreases with time according to the formula

$$M = 80(2)^{-t}$$

where t is the time in years.

a) The isotope weighs 80 grams at the start.

Show on the grid below,
how the mass of this isotope changes
over the following 4 years. 3 KU

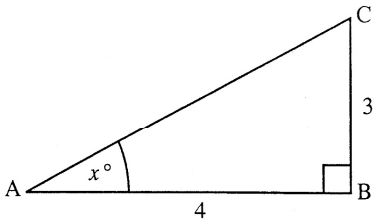


b) Calculate how many years it takes for an isotope weighing 80 grams to decrease to a weight of $\frac{5}{8}$ of a gram. 2 KU

14. Trigonometry 3 - Graphs & Equations

Graphs, triangles, maxima and minima

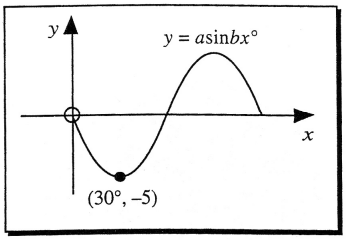
1. ABC is a right angled triangle with AB = 4 units and BC = 3 units



Prove that for the angle marked $x°$

$$\sin^2 x + \cos^2 x = 1$$ 2 KU

2. Shown is the graph of $y = a \sin bx°$



Write down the values of a and b. 2 KU

3. On a certain day the depth, D metres, of water at a fishing port, t hours after midnight, is given by the formula

$$D = 12.5 + 9.5\sin(30t)°$$

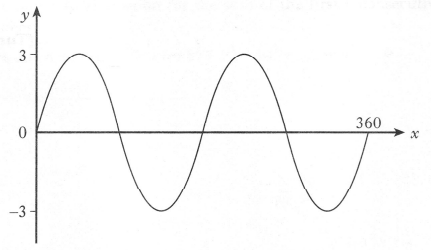
a) Find the depth of water at 1.30 pm 3 RE

b) The depth of water in the harbour is recorded each hour. What is the maximum difference in the depths of water in the harbour, over the 24 hour period ?

Show clearly all your working. 3 RE

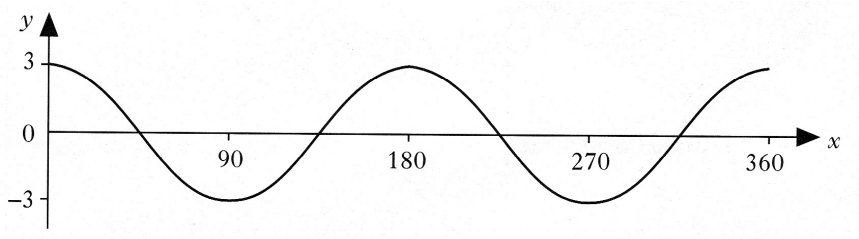
4. The diagram shows the graph of

$$y = k \sin ax°,\ 0 \le x \le 360$$



Find the values of a and k. 2 RE

5.



The diagram shows the graph of $y = a\cos bx°, \quad 0 \le x \le 360$

Find the values of a and b. 2 KU

Solving Equations

1. Solve the equation $3\tan x^o + 5 = 0$, for $0 \le x \le 360$. 4 KU

2. Solve **algebraically** the equation $2 + 3\sin x° = 0$ for $0 \le x \le 360$ 3 KU

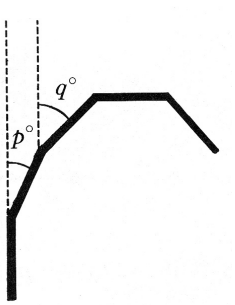
3. Solve **algebraically**, the equation $7\cos x° - 2 = 0$ for $0 \le x \le 360$ 3 KU

4. Solve **algebraically**, the equation $5\tan x - 9 = 0$, for $0 \le x \le 360$ 3 KU

5. Solve the equation $5\sin x° + 2 = 0$, *for* $0 \le x \le 360$ 3 KU

6. Solve algebraically the equation: $\tan 40° = 2\sin x° + 1$ $0 \le x \le 360$ 3 KU

7. The diagram opposite shows part of a natural crystal of topaz.

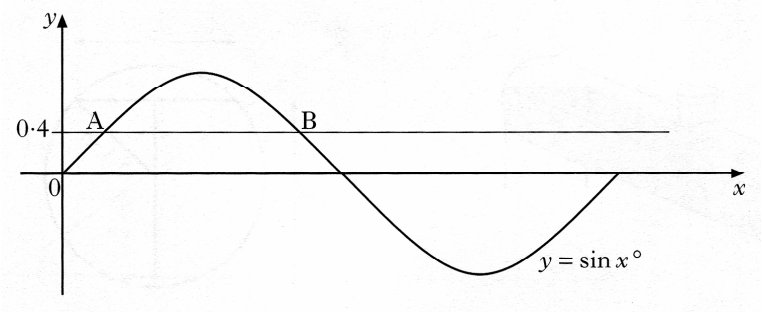


The relationship between the angles marked $p°$ and $q°$ is

$$2\tan p° = \tan q°$$

Find the value of q when p = 24. 3 KU

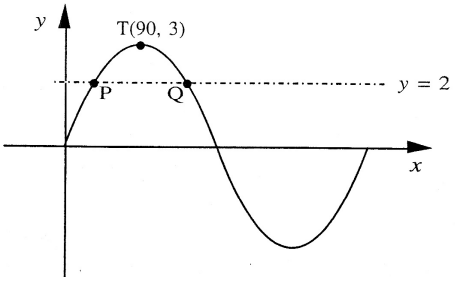
8. The diagram shows part of the graph of $y = \sin x$.



The line $y = 0.4$ is drawn and cuts the graph of $y = \sin x$ at A and B.

Find the x-coordinates of A and B. 3 RE

9. The graph shown has equation $y = a \sin bx°$.



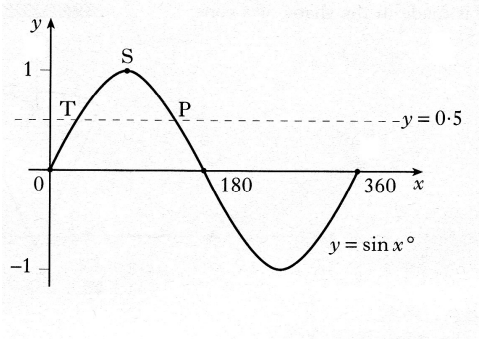
It has a maximum at the point T(90, 3).

a) Write down the values of a and b. 1 KU

Also shown in the figure is the line with equation y = 2, which meets the curve at the points P and Q.

b) Find the x-coordinate of the point Q. 3 RE

10. The diagram shows the graph of $y = \sin x°, \quad 0 \le x \le 360$

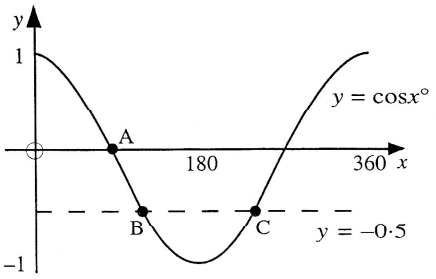


a) Write down the coordinates of point S. 1 KU

The straight line y = 0.5 cuts the graph at T and P.

b) Find the coordinates of T and P. 2 KU

11. The diagram shows the graph of $y = \cos x°, \quad 0 \le x \le 360$.

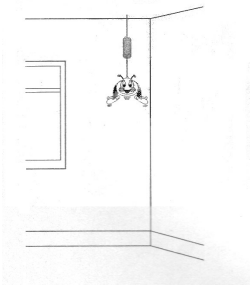


a) Write down the coordinates of point A. 1 KU

The straight line $y = -0.5$ cuts the graph at B and C.

b) Find the coordinates of B and C. 3 KU

12. A toy is hanging by a spring from the ceiling.



Once the toy is set moving, the height, H metres, of the toy above the floor is given by the formula

$$h = 1.9 + 0.3\cos(30t)°$$

t seconds after starting to move.

a) State the maximum value of H. 1 KU

b) Calculate the height of the toy above the floor after 8 seconds. 3 RE

c) When is the height of the toy first 2.05 metres above the floor? 3 RE

13. The volume of water, V millions of gallons, stored in a reservoir during any month is to be predicted by using the formula

$$V = 1 + 0.5\cos(30t)°$$

where t is the number of the month. (For January $t = 1$, February $t = 2$ …)

a) Find the volume of water in the reservoir in October. 3 RE

b) The local council would need to consider water rationing during any month in which the volume of water stored is likely to be less than 0.55 million gallons.

Will the local council need to consider water rationing?

Justify your answer. 4 RE