Vectors Homework Solutions Total = 30 Higher

Q1. $\underline{a} = \begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix}$ $\underline{b} = \begin{pmatrix} 4 \\ -3 \\ -6 \end{pmatrix}$

(i) $2\underline{a} - \underline{b} \Rightarrow \begin{pmatrix} 6 \\ 4 \\ -2 \end{pmatrix} - \begin{pmatrix} 4 \\ -3 \\ -6 \end{pmatrix} = \begin{pmatrix} 2 \\ 7 \\ 4 \end{pmatrix}$ ✓

(ii) $|2\underline{a} - \underline{b}| = \sqrt{2^2 + 7^2 + 4^2}$
$= \sqrt{69}$ ✓

$\frac{2}{2}$

Q2. $\vec{AC} = 2\vec{AB}$ $A(2,3)$ $B(-1,5)$ Find C

$\underline{a} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ $\underline{b} = \begin{pmatrix} -1 \\ 5 \end{pmatrix}$

$\vec{AB} = \underline{b} - \underline{a} = \begin{pmatrix} -1 \\ 5 \end{pmatrix} - \begin{pmatrix} 2 \\ 3 \end{pmatrix} = \begin{pmatrix} -3 \\ 2 \end{pmatrix}$ ✓

$\vec{AC} = 2\begin{pmatrix} -3 \\ 2 \end{pmatrix} = \begin{pmatrix} -6 \\ 4 \end{pmatrix}$ ✓ $\underline{c} - \underline{a} \Rightarrow \begin{pmatrix} x \\ y \end{pmatrix} - \begin{pmatrix} 2 \\ 3 \end{pmatrix} = \begin{pmatrix} -6 \\ 4 \end{pmatrix}$

$\underline{c} = \begin{pmatrix} -4 \\ 7 \end{pmatrix}$ $\underline{C = (-4, 7)}$ ✓

$\frac{3}{3}$

Q3. $A(0,-3,5)$ $B(7,-6,9)$ $C(21,-12,17)$

Show collinearity + state $AB : BC$

$\vec{AB} = \underline{b} - \underline{a} = \begin{pmatrix} 7 \\ -6 \\ 9 \end{pmatrix} - \begin{pmatrix} 0 \\ -3 \\ 5 \end{pmatrix} = \begin{pmatrix} 7 \\ -3 \\ 4 \end{pmatrix}$ ✓ $\vec{BC} = \underline{c} - \underline{b} = \begin{pmatrix} 21 \\ -12 \\ 17 \end{pmatrix} - \begin{pmatrix} 7 \\ -6 \\ 9 \end{pmatrix} = \begin{pmatrix} 14 \\ -6 \\ 8 \end{pmatrix}$ ✓

$2\vec{AB} = \vec{BC}$ ✓ $\Rightarrow$ vectors parallel and share a common point B so are collinear ✓

Ratio $\Rightarrow$ $\vec{AB} : \vec{BC} = 1:2$ ✓

$\frac{5}{5}$

Q4. $\underline{u} = 2\underline{i} - 2\underline{j} + 4\underline{k}$ $v = \underline{i} + a\underline{j} + \sqrt{7}\underline{k}$ $|u| = |v|$, find a

$u = \begin{pmatrix} 2 \\ -2 \\ 4 \end{pmatrix}$ $|u| = \sqrt{2^2 + (-2)^2 + 4^2} = \sqrt{24}$ ✓ $v = \begin{pmatrix} 1 \\ a \\ \sqrt{7} \end{pmatrix}$ $|v| = \sqrt{1^2 + a^2 + \sqrt{7}^2} = \sqrt{8 + a^2}$ ✓

$\sqrt{24} = \sqrt{8 + a^2}$ ✓ $\Rightarrow 24 = 8 + a^2 \Rightarrow a^2 = 16$ $\underline{a = \pm 4}$ ✓

$\frac{4}{4}$

Q5. $A(6,-1,9)$ $B(3,-2,11)$ $C(7,-8,14)$

Show right-angled at B

$\Rightarrow A\hat{B}C = 90^\circ \Rightarrow \vec{BA} \perp \vec{BC} \Rightarrow \vec{BA}.\vec{BC} = 0$

$\vec{BA} = \underline{a} - \underline{b} = \begin{pmatrix} 6 \\ -1 \\ 9 \end{pmatrix} - \begin{pmatrix} 3 \\ -2 \\ 11 \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \\ -2 \end{pmatrix}$ ✓ $\quad \vec{BC} = \underline{c} - \underline{b} = \begin{pmatrix} 7 \\ -8 \\ 14 \end{pmatrix} - \begin{pmatrix} 3 \\ -2 \\ 11 \end{pmatrix} = \begin{pmatrix} 4 \\ -6 \\ 3 \end{pmatrix}$ ✓

$\vec{BA}.\vec{BC} = (3)(4) + (1)(-6) + (-2)(3) = 12 - 6 - 6 = 0$ ✓

$\cos\theta = \frac{\vec{BA}.\vec{BC}}{|\vec{BA}||\vec{BC}|} = \frac{0}{|\vec{BA}||\vec{BC}|} = 0 \Rightarrow \Delta$ is right-angled, because $\cos^{-1} 0 = 90^\circ$ ✓ $\frac{4}{4}$

Q6. (a) $A(8,4,0)$ find B and C

$B(12,8,4)$ ✓ $C(16,12,2)$ ✓

(b) Angle $A\hat{B}C$

$\cos\theta = \frac{a.b}{|a||b|}$ $\quad \frac{\vec{BA}.\vec{BC}}{|\vec{AB}||\vec{BC}|}$ $\quad \vec{BA} = \begin{pmatrix} 8 \\ 4 \\ 0 \end{pmatrix} - \begin{pmatrix} 12 \\ 8 \\ 4 \end{pmatrix} = \begin{pmatrix} -4 \\ -4 \\ -4 \end{pmatrix}$ ✓

$\vec{BC} = \begin{pmatrix} 16 \\ 12 \\ 2 \end{pmatrix} - \begin{pmatrix} 12 \\ 8 \\ 4 \end{pmatrix} = \begin{pmatrix} 4 \\ 4 \\ -2 \end{pmatrix}$ ✓ or $\quad \vec{BA}.\vec{BC} = (-4)(4) + (-4)(4) + (-4)(-2)$

$= -24$ ✓

$|\vec{BA}| = \sqrt{(-4)^2 + (-4)^2 + (-4)^2} = \sqrt{48}$ ✓ $\quad |\vec{BC}| = \sqrt{4^2 + 4^2 + (-2)^2} = \sqrt{36} = 6$ ✓

$\cos\theta = \frac{-24}{\sqrt{48}(6)}$ ✓ $\quad \theta = \cos^{-1}\left(\frac{-4}{\sqrt{48}}\right)$ $\quad \theta = 125.3^\circ$ ✓ $\quad \frac{7}{7}$

Q7. $|a| = 2$ $\quad |b| = 3\sqrt{3}$

(a) (a) $a.a = |a|^2 = 4$ ✓ (b) $b.b = |b|^2 = 3\sqrt{3}(3\sqrt{3}) = 27$ ✓

(c) $a.b = |a||b|\cos\theta = |2||3\sqrt{3}|\cos 45^\circ = (2)(3\sqrt{3})\left(\frac{1}{\sqrt{2}}\right)$ ✓

$= \frac{6\sqrt{3}}{\sqrt{2}} \frac{\times\sqrt{2}}{\times\sqrt{2}} = \frac{6\sqrt{6}}{2} = 3\sqrt{6}$ ✓ $\quad \frac{4}{4}$

(b) $p = 2a + 3b$ evaluate $p.p$

$p.p = |p|^2$

$p.p = (2a+3b).(2a+3b)$ ✓

$= 4a.a + 6a.b + 6a.b + 9b.b$

$= 4a.a + 12a.b + 9b.b$ ✓

$= 4(4) + 12(3\sqrt{6}) + 9(27)$

$= 16 + 36\sqrt{6} + 243$

$= 259 + 36\sqrt{6}$

$= \underline{347.2}$ ✓ $\frac{3}{3}$