March 09, 2018

Differential Equations

A differential equation is an equation involving one or more derivatives.

 $(x^2-1)\frac{dy}{dx} + 2xy = x$ is a first order differential equation as it contains the first derivative only.

 $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 4y = 0$ is a second order differential equation as it contains the second derivative.

Differential Equations

If we have a function y = f(x), we can verify that the function satisfies a particular differential equation.

For example: Verify that $y = Ae^{5s} + Be^{-2s}$ where A, B are constants satisfies the differential equation $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} - 10y = 0$ $y = Ae^{5x} + Be^{-2x}$ $\frac{dy}{dx} = 5Ae^{5x} - 2Be^{2x}$ $\frac{d^2y}{dx^2} = 25Ae^{5x} + 4Be^{2x}$ $25Ae^{5x} + 4Be^{2x} - 3(5Ae^{5x} - 2Be^{2x}) - 10(Ae^{5x} + Be^{2x})$ = $25Ae^{5x} + 4Be^{2x} - 15Ae^{5x} + 6Be^{3x} - 10Ae^{5x} - 10Be^{2x}$ = $10Ae^{5x} - 10Ae^{5x} + 10Be^{2x} - 10Be^{2x}$ = $0 \xrightarrow{-2} \text{ solitifies the equation}$ 1

General solution of differential equations

If a first order differential equation can be written in the form

f(y)dy = g(x)dx

where f(y) is a function of y only and g(x) is a function of x only, we say that we have separated the variables x and y.

The general solution of the differential equation is then found by integrating each side with respect to the appropriate variable.

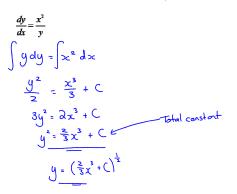
 $\int f(y)dy = \int g(x)dx$

General solution of differential equations 1. Find the general solution of the differential equation



General solution of differential equations

2. Find the general solution of the differential equation



General solution of differential equations

3. Find the general solution of the differential equation

$$e^{4y}\frac{dy}{dx} - x = 2$$

1

$$e^{4y} \frac{dy}{dx} = \lambda + x$$

$$e^{4y} \frac{dy}{dx} = \lambda + x$$

$$\int e^{4y} \frac{dy}{dy} = \int (2+x) dx$$

$$\frac{1}{4} e^{4y} = 2x + \frac{x^2}{2} + C$$

$$e^{4y} = 8x + 2x^2 + C$$
Take the loge of both sides

$$4y = ln |8x+2x^2+c|$$

 $y = 4ln |8x+2x^2+c|$

General solution of differential equations

4. Find the general solution of the differential equation

$$\frac{dy}{dx} = \frac{1}{16 + 4x^2}$$

$$\int dy = \frac{1}{(6 + 4x^2)} dx$$

$$\int Idy = \int \frac{1}{4(4 + x^2)} dx$$

$$\int Idy = \frac{1}{4} \int \frac{1}{2^2 + x^2} dx$$

$$y = \frac{1}{4} \left[\frac{1}{2} \tan^{-1} \left(\frac{x}{2} \right) \right] + C$$

$$y = \frac{1}{4} \left[\frac{1}{2} \tan^{-1} \left(\frac{x}{2} \right) \right] + C$$

General solution of differential equations

5. Find the general solution of the differential equation

$$\frac{dy}{dx} = \frac{2}{\sin y}$$
Siny dy = $2dx$
 $\int siny dy = \int 2dx$
 $-\cos y = 2x + C$
 $\cos y = -2x + C$
 $y = \cos^{-1}(-2x+C)$

General solution of differential equations

6. Find the general solution of the differential equation

$$\frac{dy}{dx} = 2x\sqrt{9-y^2}$$

$$dy = 2x\sqrt{9-y^2} \quad d>c$$

$$\int \frac{1}{\sqrt{9-y^2}} \quad dy = \int \partial x \quad d>c$$

$$\sin^{-1}\left(\frac{y}{3}\right) = x^2 + C$$

$$\frac{y}{3} = \sin(x^2 + C)$$

$$y = 3\sin(x^2 + C)$$

General solution of differential equations

7. Find the general solution of the differential equation

$$(1+x^{2})y^{2}\frac{dy}{dx} = 1$$

$$\int y^{2} dy = \int \frac{1}{1+x^{2}} dx$$

$$\frac{y^{3}}{3} = \tan^{-1}(x) + C$$

$$y^{3} = 3\tan^{-1}(x) + C$$

$$y = (3\tan^{-1}(x) + C)^{\frac{1}{3}}$$

General solution of differential equations

We can use properties of logs and exponents to help us simplify general solutions.

Examples:

1. Find the general solution of the differential equation

$$\frac{dy}{dx} = 4y$$

$$dy = 4y dx$$

$$\int \frac{1}{y} dy = 4y dx$$

$$l_{\Lambda} |y| = 4x + C$$

$$e^{\Lambda} = e^{\Lambda}$$

$$y = e^{4x+C}$$

$$y = e^{4x+C}$$

$$e^{4x+C}$$

$$y = e^{4x+C}$$

$$y = e^{4x}$$

General solution of differential equations

2. Find the general solution of the differential equation

$$\frac{dy}{dx} = \frac{y+2}{x+1} \qquad (x+i)dy = (y+2)dx \qquad i = \frac{1}{y+2} dy = \frac{1}{x+1} dx$$

$$\int \frac{1}{y+2} dy \cdot \int \frac{1}{x+1} dx$$

$$\ln |y+2| = \ln |x+1| + C \qquad \ln |x+1| + C$$

$$e^{n} \qquad e^{n} \qquad e^{n} \qquad e^{n|x+1|} e^{c}$$

$$y+2 = (x+1).e^{c} \qquad = e^{n|x+1|} e^{c}$$

$$y = (x+1).e^{c} - 2 \qquad e^{n|x+1|} e^{c}$$

$$y = (x+1).e^{c} - 2 \qquad e^{n|x+1|} e^{c}$$

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3. Find the general solution of the differential equation

$$\frac{dy}{dx} = \frac{y}{2x-1}$$

$$\int \frac{1}{y} dy = \int \frac{1}{2x-1} dz$$

$$\int \frac{1}{y} dy = \int \frac{1}{2x-1} dz$$

$$\int \frac{1}{y} dy = \frac{1}{2} \ln |ax-1| + C$$

$$\int \frac{1}{y} \ln |ax-1| + C$$

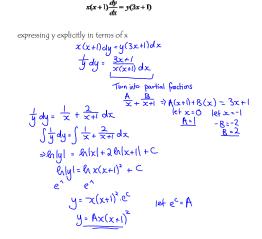
Examples:
1. (a) Express
$$\frac{2x+3}{x(x+1)}$$
 in partial fractions
 $\frac{A}{x} + \frac{B}{x+1} = \frac{2x+3}{x(x+1)}$
 $\frac{A}{x} + \frac{B}{x+1} = \frac{2x+3}{x(x+1)}$
A(x+1) + Bx = 2x+3
B = -1
(b) Hence find the general solution of the differential equation
 $x(x+1)\frac{d}{dx} = y(2x+3)$
expressing y explicitly in terms of x
 $\int \frac{1}{y} dy = \int \frac{3x}{x(x+1)} dx$
 $\int \frac{1}{y} dy = \int \frac{3x}{x(x+1)} dx$
 $\int \frac{1}{y} dy = \int \frac{3x}{x(x+1)} dx$
 $A(x+1) + Bx = 2x+3$
 $\int \frac{1}{y} dy = \int \frac{3x}{x(x+1)} dx$
 $\int \frac{1}{y} dy = \int \frac{3x}{x(x+1)} dx$

Differential equations with partial fractions

Differential equations with partial fractions.
2. (a) Express
$$\frac{x+1}{(2x+1)}$$
 in partial fractions.
(b) Hence find the general solution of the differential equation
 $\frac{d'}{dt} = \frac{x(x+1)}{x(2x+1)}$
expressing y explicitly in terms of x
(a) $\frac{x+1}{\chi(2x+1)} = \frac{A}{\chi} + \frac{B}{2x+1} = \frac{1}{\chi} - \frac{1}{2\chi+1}$
 $A(2x+1) + B(X) = x+1$
 $A(2x+$

Differential equations with partial fractions 3. Find the general solution of the differential equation

$x(x+1)\frac{dy}{dx} = y(3x+1)$



Particular Solutions of differential equations If we are given initial conditions for x and y, we can find the particular solution of a differential equation.

Examples: 1. Find the particular solution of the differential equation

$$y^2 \frac{dy}{dx} = 4x^2 + 1$$

given that y = 2 when x = 1, expressing y in terms of
$$u^2 d\mu = (\mu v^2 + 1) dv$$

$$\int y^{2} dy = \int (y^{2} + 1) dx$$

$$\int y^{2} dy = \int (y^{2} + 1) dx$$

$$=^{2} \frac{y^{3}}{3} = \frac{4x^{3}}{3} + x + C$$

$$=) \quad y^{3} = 4x^{3} + x + C$$

$$y^{*2} \quad \text{when } x = 1$$

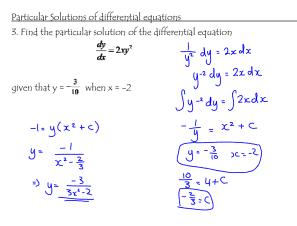
$$a^{3} = 4 + 3 + C$$

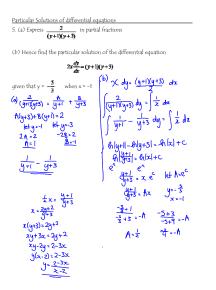
$$g^{*2} \quad \text{when } x = 1$$

$$a^{3} = 4 + 3 + C$$

$$C = 1$$

$$y = (4x^{5} + 3x + 1)$$





Particular Solutions of differential equations
Past Paper Question:

2017	5 marks
2017	

Solve
$$\frac{dy}{dx} = e^{2x}(1+y^2)$$
 given that when x = 0, y = 1. Express y in terms of x

$$dy = e^{2x} (1+y^{2}) dx$$

$$\int \frac{1}{1+y^{2}} dy = \int e^{2x} dx$$

$$\tan^{4}(y) = \frac{1}{2} e^{2x} + C \qquad x y$$

$$y = \tan(\frac{1}{2}e^{2x} + C) \qquad (0,1)$$

$$i = \tan(\frac{1}{2} + C)$$

$$\tan(\frac{1}{2}e^{2x} + C) \qquad y = \tan(\frac{1}{2}e^{2x} + \frac{\pi}{4} - \frac{1}{2})$$

$$\frac{\pi}{4} = \frac{1}{2} + C \qquad y = \tan(\frac{1}{2}e^{2x} + \frac{\pi}{4} - \frac{1}{2})$$

Particular Solutions of differential equations
4. Find the particular solution of the differential equation

$$(x+4)\frac{dy}{dx} = y-1$$

$$(x+4)\frac{dy}{dx} = (y-1)\frac{dy}{dx}$$
given that $y = 7$ when $x = -1$ $\int \frac{1}{y-1} \frac{dy}{dy} = \frac{1}{x+4} \frac{dx}{dx}$

$$= \frac{\ln|y-1|}{\ln|x} \frac{\ln|x}{\ln|y|} + C$$

$$e^{A} e^{A}$$

$$y-1 = (x+4) e^{C} e^{A} \ln|x+4| + C$$

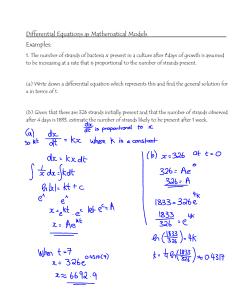
$$y-1 = A(x+4) + 1 = \frac{y-7}{2x-1}$$

$$7 = A(-(+4) + 1)$$

$$7 = 3A+1$$

$$3A = 6 \qquad y = 2(x+4)+1$$

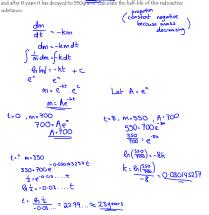
$$y = 2x+9$$





Examples:

2. The mass m(grams) of a radioactive substance at time (Kyam) decreases at a rate which is proportional to the mass at that time. The half-life for a radioactive substance is the time taken for half of the mass to decay. The original halfs of a radioactive substance is 700 grams and after 8 years it has decayed to 500 grams. Calculate the half-life of this radioactive substance substance.



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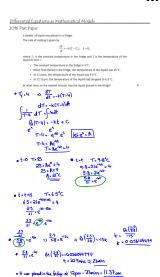
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Further Differential Equations

So far we have only been working with non-linear first order differential equations that are separable.

What about this type of equation?

 $\frac{dy}{dx} + 2xy = 6e^{2x}$

Further Differential Equations

A first order differential equation where the variables cannot be separated is known as a first order linear differential equation and is generally of the form

$\frac{dy}{dx} + P(x)y = Q(x)$

We need to perform some manipulation to get the equation into a form that we can work with. The method we use is called the integrating factor method.

 $I(x) = e^{\int r(x)dx}$ is known as the integrating factor.

Earther Differential Equations Examples: 1. Find the general solution of the differential equation $\frac{dy}{dx} + y = 6e^{2x}$ $P(x) = 1 \qquad T(x) = e^{\sum 1dx} = e^{x}$ $I(x) = e^{\int 1dx} = e^{x}$ $I(x) = e^{\int 1dx} = e^{x}$ $I(x) = e^{\int 1dx} = e^{\sum 1dx}$ $I(x) = e^{\int 1dx} = e^{\sum 1dx}$ $I(x) = e^{\int 1dx} = e^{\sum 1dx}$

$$\int \frac{d}{dx} (e^{x} y) = be^{3x}$$

$$\int \frac{d}{dx} (e^{x} y) = \int be^{3x} dx$$

$$e^{x} y = \partial e^{3x} + C$$

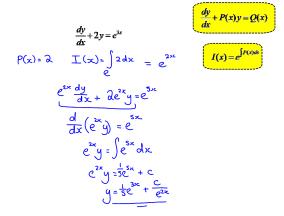
$$y = \partial e^{3x} + C$$

$$\int \frac{d}{dx} e^{x} y = \partial e^{3x} + C$$

Further Differential Equations

Examples:

2. Find the general solution of the differential equation

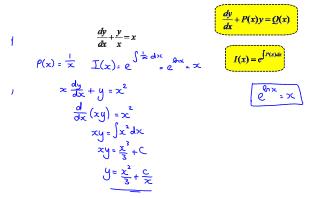


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Examples:

3. Find the general solution of the differential equation



Further Differential Equations Examples:

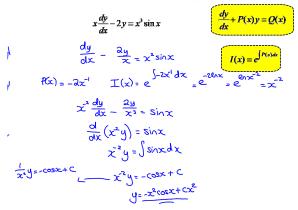
4. Find the general solution of the differential equation

 $\frac{dy}{dx} + P(x)y = Q(x)$ $\frac{dy}{dx} = \frac{xy+4}{x^2}$ $\frac{dy}{dx} = \frac{xy}{x^2} + \frac{4}{x^2}$ $I(x) - e^{\int P(x) dx}$ $P(x) = -\frac{1}{x} \qquad I(x) = e^{\int_{-\frac{1}{x}}^{x} dx} = e^{e_{x}x'} = \frac{1}{x}$ t $\frac{1}{x}\frac{dy}{dx} - \frac{1}{x^2}y = \frac{4}{x^3}$ $\frac{d}{dx}(\frac{1}{x}y) = \frac{4}{x^3}$ $\frac{1}{2} y = \int 4x^{2} dx$ $\frac{1}{2} y = -\partial x^{2} + C$ $y = -\partial x^{2} + C$

Further Differential Equations

Examples:

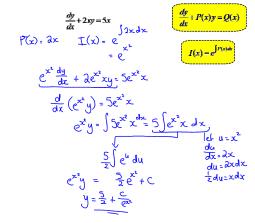
5. Find the general solution of the differential equation



Further Differential Equations

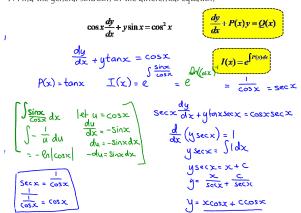
Examples:

6. Find the general solution of the differential equation



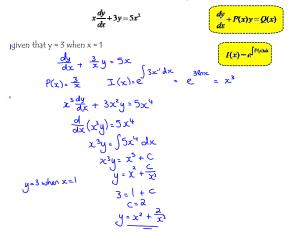


7. Find the general solution of the differential equation



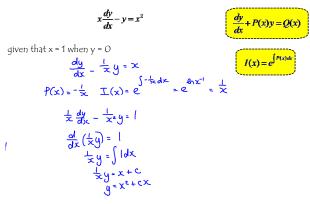
Further Differential Equations – Particular Solutions

1. Find the particular solution for the differential equation



Further Differential Equations - Particular Solutions

2. Find the particular solution for the differential equation



When to use integration by parts:

- Always try substitution first.
- If neither factor is related to the derivative of the other, use integration by parts.

Second Order Differential Equations

These are equations of the form

$$a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = f(x)$$

where a, b and c are constants, $a \neq 0$ and f(x) is a function of x only. If f(x) = 0, the differential equation is said to be homogeneous.

If $f(x) \neq 0$, the differential equation is said to be non-homogeneous.

Homogeneous 2nd order differential equations

 $a\frac{d^2y}{dr^2}+b\frac{dy}{dr}+cy=0$

As these equations will involve a 2nd derivative, the general solution will have 2 arbitary constants.

We find the general solution by considering a solution of the form $y = e^{mx}$ $y = e^{mx}$ $\frac{dy}{dx} = me^{mx}$ $\frac{d^2y}{dx^2} = m^2 e^{mx}$ $am^2e^{mx} + bme^{mx} + ce^{mx} = 0$ $\div e^{mx}$ both $am^2 + bm + c = 0$ $\div constants$

Homogeneous 2nd order differential equations

 $am^2 + bm + c = 0$ is known as the auxilliary equation and can be factorised to solve for m.

The nature of the roots can be found by using $b^2 - 4ac$.

 $b^2 - 4ac > 0$ 2 real roots $y = Ae^{m_k x} + Be^{m_k x}$ $b^2 - 4ac = 0$ 2 equal roots $y = e^{int}(A + Bx)$ $b^2 - 4ac < 0$ 2 complex roots $y = e^{px} (A\cos qx + B\sin qx)$ $M = P \pm Q$

https://www.khanacademy.org/math/differential-equations For further reading on proofs!

Homogeneous 2nd order differential equations Examples:

Find the general solution of the 2nd order differential equation

1.
$$\frac{d^{2}y}{dx^{2}} + 2\frac{dy}{dx} - 8y = 0$$

(m² + 2m - 8 = 0
(m + 4 x m - 2) = 0
(m = -4, m = 2)

y= e^{mx} y= e^{-4x} y= e^{2x} y= Ae^{-4x} y= Be^{2x}

General Solution & y=Ae^{4x}+Be^{2x}

Homogeneous 2nd order differential equations Examples:

Find the general solution of the 2nd order differential equation

2.
$$\frac{d^{2}y}{dx^{2}} - 6\frac{dy}{dx} + 9y = 0$$

$$m^{2} - 6m + 9 = 0$$

$$(m - 3)(m - 3) = 0$$

$$y = Ae^{3x}$$

$$y = Bx e^{3x}$$

$$y = Ae^{3x} + Bx e^{3x}$$

$$g_{enveral} = Solution^{2}$$

$$y = \frac{Ae^{3x} + Bx e^{3x}}{e^{3x}(A + Bx)}$$

Homogeneous 2nd order differential equations

Examples:

Find the general solution of the 2nd order differential equation

$$3. \frac{d^{2}y}{dx^{2}} + 4\frac{dy}{dx} + 13y = 0$$

$$m^{2} + 4m + 13 = 0$$

$$m = -\frac{b^{\pm}\sqrt{b^{2} - 4ac}}{2a} = -\frac{4^{\pm}\sqrt{16 - 52}}{2} = -\frac{4^{\pm}\sqrt{-36}}{2}$$

$$m = -\frac{4^{\pm}b^{2}}{2} = -\frac{2^{\pm}3}{2}$$

$$y = \frac{e^{2x}(A\cos 3x + B\sin 3x)}{2}$$

Homogeneous 2nd order differential equations

Homogeneous 2nd order differential equations

Find the particular solution of the second order differential equation

 $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 5y = 0 \quad \text{given that when } x = 0, y = 2 \text{ and } \frac{dy}{dx} = 1$

y=e^{**}Acosx + e^{**}Bsinx 2=e[°]Acos0 + e[°]Bsin0

 $\frac{dy}{dx} = e^{2x} (-A \sin_{1}x) + A \cos_{2}x (2e^{2x})$ $+ Be^{2x} (\cos x) + \sin x (2Be^{2x})$

l=-Ae°sin0+Acos0(Ze°)

+ Be° (cos0) + 2Be° sin0

2 = A

1= 2A+B 1=4+B 8=-3

Particular Solutions

 $m^2 - 4m + 5 = 0$

62-4ac 20 => no real roots

 $m = -(-4) + \sqrt{(-4)^2 - 4(1)(5)}$

 $y = e^{2x} (Arosx + Bsinx)$

 $y = e^{2x} (2\cos x - 3\sin x)$

2(1) $m = \frac{4 \pm \sqrt{-4}}{2} = \frac{4 \pm 2i}{2} = \frac{1}{2} = \frac{1}{2}$

Examples:

3.

Particular Solutions

Examples:

Find the particular solution of the second order differential equation

1.
$$\frac{d^{2}y}{dx^{2}} - 4\frac{dy}{dx} + 3y = 0 \quad \text{given that when } x = 0, y = 1 \text{ and } \frac{dy}{dx} = 5$$

$$m^{2} - 4m + 3 = 0 \qquad \text{When } x = 0, y = 1 \text{ and } \frac{dy}{dx} = 5$$

$$m^{2} - 4m + 3 = 0 \qquad \text{When } x = 0, y = 1 \text{ and } \frac{dy}{dx} = 5$$

$$m^{2} - 4m + 3 = 0 \qquad \text{When } x = 0, y = 1 \text{ and } \frac{dy}{dx} = 5$$

$$m^{2} - 4m + 3 = 0 \qquad \text{When } x = 0, y = 1 \text{ and } \frac{dy}{dx} = 5$$

$$m^{2} - 4m + 3 = 0 \qquad \text{When } x = 0, y = 1 \text{ and } \frac{dy}{dx} = 5$$

$$m^{2} - 4m + 3 = 0 \qquad \text{When } x = 0, y = 1 \text{ and } \frac{dy}{dx} = 5$$

$$m^{2} - 4m + 3 = 0 \qquad \text{When } x = 0, y = 1 \text{ and } \frac{dy}{dx} = 5$$

$$y = \frac{Ae^{x} + Be^{3x}}{M^{2} + Be^{3x}} \qquad \frac{dy}{dx} = Ae^{x} + 3Be^{x}$$

$$5 = Ae^{x} + 3Be^{x}$$

$$4 = 2B B = A$$

Homogeneous 2nd order differential equations

Particular Solutions

Examples:

Find the particular solution of the second order differential equation

2.
$$\frac{d^{2}y}{dt^{2}} + 4\frac{dy}{dt} + 4y = 0 \quad \text{given that when } x = 0, y = 1 \text{ and } \frac{dy}{dt} = 2$$

$$m^{2} + 4m + 4 = 0 \quad |z = Ae^{0} + B(0)e^{0}$$

$$m = -2 \quad Ae^{-2t} + Be^{-2t}$$

$$m = -2 \quad Ae^{-2t} + Be^{-2t}$$

$$m = -2 \quad Ae^{-2t} + Be^{-2t}$$

Non-Homogeneous 2nd order differential equations

These are equations of the form

$$a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = f(x)$$

where a, b and c are constants and $f(x) \neq 0$. Q×0

- First treat it as if it were homogeneous and equate to zero to find the complementary function.
- Find a particular integral y_p (it will be in a similar form to f(x)).
- General Solution = Complementary Function + Particular Integral

 $y = y_c + y_p$

Identifying the particular integral

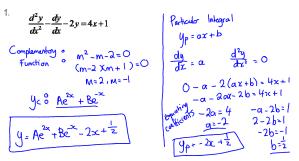
- Linear, e.g. f(x) = 2x + 1 $y_p = ax + b$
- Quadratic, e.g. $f(x) = x^2 1$ $y_p = ax^2 + bx + c$
- e, e.g. f(x) = 4e^{2x}
- Trigonometric, e.g. $f(x) = 2\sin x + \cos x$ $y_p = a\sin x + b\cos x$

 $y_p = Ae^{2x}$

- or e.g. f(x) = 3sin2x
- $y_p = asin2x + bcos2x$ include because 3sin2x = 3sin2x + 0cos2x

<u>Non-Homogeneous 2nd order differential equations</u> Examples:

Find the general solution of the 2nd order differential equation



Non-Homogeneous 2nd order differential equations Examples:

Find the general solution of the 2nd order differential equation

2.

$$\frac{d^{2}y}{dx^{2}} - 4\frac{dy}{dx} + 4y = 2x^{2} + 1$$

$$M^{2} - 4m + 4 = 0$$

$$(m - 2 \times m - 2) = 0$$

$$\frac{m = 2}{y_{c}}$$

$$\frac{y_{c}}{x} - \frac{Ae^{2x} + Bxe^{2x}}{y_{c}} + \frac{1}{2}x^{2} + x + 1$$

$$\frac{y_{e}}{x} + \frac{bxe^{2x}}{x} + \frac{1}{2}x^{2} + x + 1$$

$$\frac{y_{e}}{x} + \frac{bxe^{2x}}{x} + \frac{1}{2}x^{2} + x + 1$$

$$\frac{y_{e}}{x} + \frac{bxe^{2x}}{x} + \frac{1}{2}x^{2} + x + 1$$

$$\frac{y_{e}}{x} - \frac{a^{4}y}{x} + \frac{bxe^{2x}}{x} + \frac{1}{2}x^{2} + x + 1$$

$$\frac{y_{e}}{x} - \frac{a^{4}y}{x} + \frac{bxe^{2x}}{x} + \frac{1}{2}x^{2} + x + 1$$

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Non-Homogeneous 2nd order differential equations Examples:

Find the general solution of the 2nd order differential equation

^{3.}
$$\frac{d^{2}y}{dx^{2}} + 2\frac{dy}{dx} - 3y = 10e^{2x}$$

$$y_{c} \otimes m^{2} + 2m - 3 = 0$$

$$(m + 3) (m - 1) = 0$$

$$\frac{m^{2-3}(m^{2})}{(m + 3)(m^{2} - 1)^{2}}$$

$$y_{c} \otimes Ae^{-3x} + Be^{\infty}$$

$$y = \frac{h}{c}^{-5x} + Be^{\infty} + Ae^{2x}$$

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<u>Non-Homogeneous 2nd order differential equations</u> Examples:

Find the general solution of the 2nd order differential equation

4.
$$\frac{d^{2}y}{dx^{2}} + 16y = e^{-2x}$$

$$y_{c} \circ m^{2} + 16y = e^{-2x}$$

$$y_{c} \circ m^{2} - 16$$

$$m^{2} - 16$$

$$m^{2} - 16$$

$$M^{2} - 16$$

$$y_{c} \circ e^{\infty}(Asin4x + Bcos4x)$$

$$= Asin4x + Bcos4x$$

$$y = Asin4x + Bcos4x$$

$$y = Asin4x + Bcos4x + \frac{1}{20}e^{-2x}$$

$$y = Asin4x + Bcos4x + \frac{1}{20}e^{-2x}$$

Non-Homogeneous 2nd order differential equations Examples:

Find the general solution of the 2nd order differential equation

5.
$$\frac{d^{2}y}{dx^{2}} - \frac{4dy}{dx} + 3y = 10\sin x$$

$$y = y + y = 9$$

$$y_{c} = \frac{4dy}{dx} + 3y = 10\sin x$$

$$y = y + y = 9$$

$$y_{c} = \frac{4dy}{dx} + 3y = 10\sin x$$

$$y = 0$$

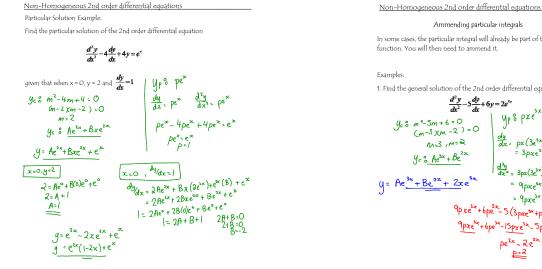
$$y = -\frac{4dy}{dx} + 3y = 10\sin x$$

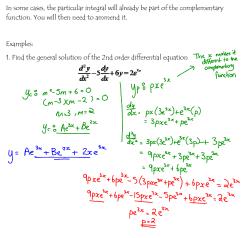
$$y = 0$$

$$y = -\frac{4dy}{dx} + 3y = 10\sin x$$

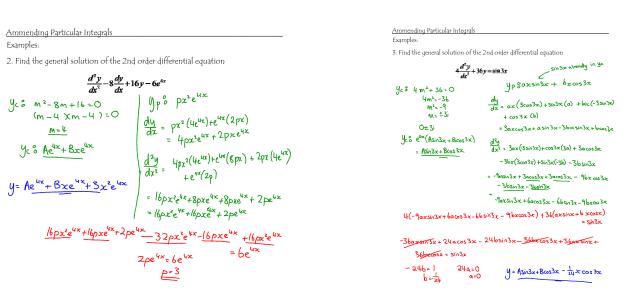
$$y = -\frac{4dy}{dx} + 3y = 10\sin x$$

$$y = -\frac{4dy}{dx} + \frac{2}{3}\cos x + \frac{4}{3}\cos x +$$





Ammending particular integrals



Ammending Particular Integrals

Examples:

2. Find the general solution of the 2nd order differential equation