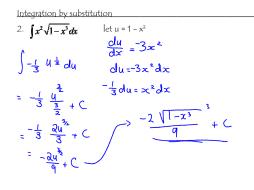
Integration by substitution

We use substitution to integrate when we have more than one fucntion e.g. products or quotients that are difficult to simplify. The function to be replaced must be similar to the other function when differentiated.

(You will be given the substitution)

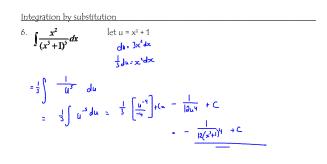
Examples: Integrate the following

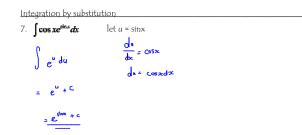
 $\int \mathbf{x} (\mathbf{x}^2 + \mathbf{i})^3 d\mathbf{x} \qquad \text{Let } \mathbf{u} = \mathbf{x}^2 + \mathbf{1}$ $\int \frac{du}{d\mathbf{x}} = 2\mathbf{x}$ $\int \frac{1}{2} \mathbf{u}^5 d\mathbf{u} \qquad d\mathbf{u} = 2\mathbf{x} d\mathbf{x}$ $\left[\frac{1}{2} \frac{\mathbf{u}^6}{6} + C\right]$ $\int x(x^2+1)^5 dx$ $= \frac{4^{6}}{12}$ $= (x^{2}+1)^{6} + C$



Integration by substitution	Integration by substitution
3. $\int \sin^3 x \cos x dx \qquad et u = \sin x$ $\int u^3 du \qquad $	4. $\int \frac{x}{3x^2+1} dx \qquad et u = 3x^2+1 \\ \frac{du}{dx} = 6x \qquad \int \frac{x}{1} \cdot \frac{1}{3x^2+1} dx$ $= \int \frac{1}{6} \cdot \frac{1}{4} du = \frac{1}{6} du = x dx$ $= \frac{1}{6} \ln u + C$ $= \frac{1}{6} \ln 3x^2+1 + C$

Integration by substitution		
$\int \frac{\sin 2x}{\sqrt{1+\cos 2x}} dx$	let u = 1 + cos2x $\frac{du}{dx} = -2\sin^2x$	
, C , I	du = - 2 sin 22cdx	
=- <u>1</u> <u>1</u> <u>1</u> <u>1</u> <u>1</u> <u>1</u> <u>1</u> <u>1</u> <u>1</u>	- Edu = storada	
$=-\frac{1}{2}\int U^{-\frac{1}{2}} du$		
$= -\frac{1}{2} \left[\frac{u^2}{2} \right] + C$	$= -u^{\frac{1}{2}} + c = -\sqrt{1+c\sigma^2 x} + C$	





Integration by substitution 8. $\int x(x+1)^3 dx$ let u = x + 1 $\frac{du}{dx} = 1$ $\frac{du}{du} = 1 dx$ ∫ (u-1) u³ du ∫ u⁴-u³ du u = >< + | u - 1 = ∞ $= \frac{u^5}{5} - \frac{u^4}{5} + C$ $(x+1)^{2} - (x+1)^{4} + C$

Integration by substitution
9.
$$\int \frac{x+1}{\sqrt{x-2}} dx \qquad |et u = x-2$$

$$\int \frac{du}{dx} = 1$$

$$\int \frac{u+2+1}{u^{\frac{1}{2}}} du \qquad \frac{u=x-2}{(x=u+2)}$$

$$= \int \frac{u+3}{u^{\frac{1}{4}}} du = \int \frac{u}{u^{\frac{1}{4}}} + \frac{3}{u^{\frac{1}{4}}} du = \int u^{\frac{1}{4}} + 3u^{-\frac{1}{4}} du$$

$$\frac{2u^{\frac{3}{4}}}{3} + \frac{3u^{\frac{1}{4}}}{\frac{1}{2}} + C$$

$$= \frac{2}{3}u^{\frac{3}{4}} + 6u^{\frac{1}{4}} + C$$

$$= \frac{2}{3}(x-2)^{\frac{3}{4}} + 6(x-2)^{\frac{1}{4}} + C$$

Integration by substitution 10. $\int x^{3} \sqrt{x^{2} + I_{d}x}$ let $u = x^{2} + 1$ $\frac{1}{2} \int (u-1)(u)^{\frac{1}{2}} du$ $\frac{du}{dx} = 2x dx$ $\frac{1}{2} \int u^{\frac{3}{2}} - u^{\frac{1}{2}} du$ $\frac{(1 + 2x)^{2} + 1}{(1 + 2x)^{2} + (1 + 1)^{2}}$ $= \frac{1}{2} \left[\frac{u^{\frac{5}{2}}}{\frac{1}{2}} - \frac{u^{\frac{1}{2}}}{\frac{1}{2}} \right] + C$ $= \frac{1}{2} \left[\frac{2u^{\frac{5}{2}}}{\frac{1}{2}} - \frac{2u^{\frac{3}{2}}}{\frac{1}{2}} \right] + C$ $= \frac{u^{\frac{5}{2}}}{2} - \frac{u^{\frac{3}{2}}}{\frac{1}{2}} + C$ $= \frac{u^{\frac{5}{2}}}{2} - \frac{u^{\frac{3}{2}}}{\frac{1}{2}} + C$

Evaulating definite integrals - Integration by substitution

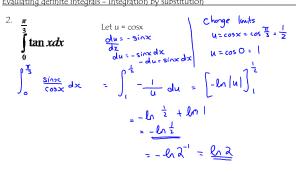
When evaluating definite integrals using substitution, we don't have to change back the original 'x' provided we change the limits at the point of substitution.

Examples: Evaluate the following

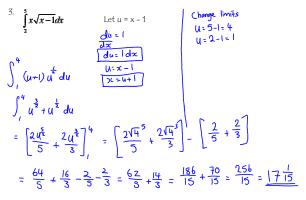
 $\int_{0}^{\infty} x^{2} (x^{3} + 1)^{3} dx \qquad \text{Let } u = x^{4} + 1$ $= \frac{1}{4} \int_{1}^{2} u^{3} du \qquad du^{2} 4 \chi^{3} dx$ $= \frac{1}{4} \int_{1}^{2} u^{4} du = x^{3} dx$ $=\frac{1}{4}\left[\frac{2^{4}}{4}\right]-\frac{1}{4}\left[\frac{1^{4}}{4}\right]=1-\frac{1}{16}\cdot\frac{15}{16}$

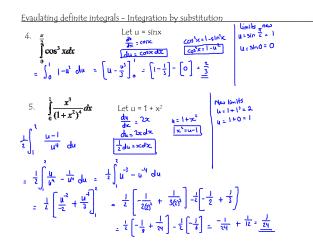
Changing limits U = 2c⁴ + 1 U = 1⁴ + (= 2 U = 0⁴ + |= 1

Evaulating definite integrals - Integration by substitution



Evaulating definite integrals - Integration by substitution





Evaulating definite integrals - Integration by substitution

Let u = 1 + x²

 $\int_{0}^{1} \frac{x^{3}}{(1+x^{2})^{4}} dx$ 5.

Integration by substitution - Using Trig. Identities

Need to remember:

 $1 + \tan^2 x = \sec^2 x$

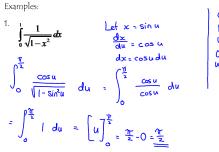
 $\cos 2x = \cos^2 x - \sin^2 x = 1 - 2\sin^2 x = 2\cos^2 x - 1$

sin2x = 2sinxcosx

2cos 2x - 1 = cos 2x 2 cos 2 u = cos 2 4 + 1

 $sin^2x + cos^2x = 1$

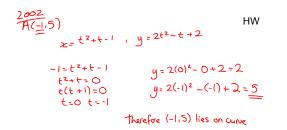
Integration by substitution - Using Trig. Identities

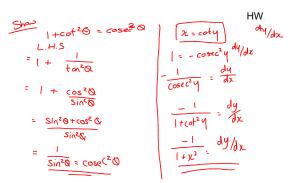


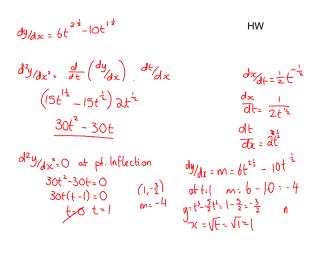
Change limits $\int_{0}^{1} z \sin 4 = \sin 4$ $\int_{0}^{1} z \sin^{-1}(1) = \frac{\pi}{2}$ $0 = \sin N$ $N = \sin^{-1}(0) = 0$



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Integration by substitution - Using Trig. Identities

Examples:

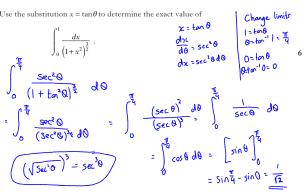


Integration by substitution - Using Trig. Identities

Examples:

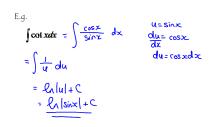
2014 Q12.

Use the substitution $x = \tan \theta$ to determine the exact value



Integration by substitution - A special case

Sometimes you will come across a fraction where the top is the derivative of the bottom.



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Integration by substitution - A special case

$$\int \frac{f'(x)}{f(x)} dx = \ln |f(x)| + C$$

Examples:

- $\int \frac{2e^{2x}}{1+e^{2x}} dx = |x||+e^{2x} + C$
- 2. $\int \frac{18x^2 4x + 10}{6x^3 2x^2 + 10x + 7} dx$ $= \frac{2}{3} \left| 6x^3 2x^2 + 10x + 7 \right| + C$

Integration by substitution – A special case

Sometimes we may need to manipulate or take out a common factor first.

^{3.}
$$\int \frac{12x+8}{3x^2+4x-8} dx = 2 \int \frac{6x+4}{3x^2+4x-8} dx$$

= 28/3x2+4x-8+C