Related Rates of Change

This topic is another application of the chain rule. We can solve problems in context with more than two variables. Given that y is a function of x and that x and y are both functions of u, the chain rule states that

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

Examples:

1. The radius of a circular ripple is increasing at a rate of 30cm per second. How fast is the area increasing when the radius is 50cm?

$\frac{dA}{dt} = \frac{dR}{dt} \times \frac{dA}{dR}$

$A = \pi r^2$

$\frac{dA}{dR} = 2\pi r$

$\frac{dR}{dt} = 30 \quad \frac{dA}{dR} = 2\pi r$

$\frac{dA}{dt} = 30 \times 2\pi r = 60\pi r$

when $r = 50$

$\frac{dA}{dt} = 60\pi \times 50 = \underline{\underline{3000\pi\,cm^2/sec}}$

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2. The volume of a spherical balloon is increasing at a rate of $0.05m^3/s$. How fast is the surface area increasing when the radius is 0.04m?

Volume sphere $= \frac{4}{3}\pi r^3$ Surface Area $= 4\pi r^2$

$\frac{dA}{dt} = \frac{dV}{dt} \times \frac{dA}{dr} \times \frac{dr}{dV}$

$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dr} = 4\pi r^2$

$\frac{dr}{dV} = \frac{1}{4\pi r^2}$

$\frac{dV}{dt} = 0.05 \quad \frac{dA}{dr} = 8\pi r$

$\frac{dA}{dt} = 0.05 \times 8\pi r \times \frac{1}{4\pi r^2}$

$= \frac{0.1}{r}$ at $r = 0.04$

$\frac{dA}{dt} = \frac{0.1}{0.04} = \underline{\underline{2.5\,m^2/second}}$

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3. A cube has edge of length x cm and is expanding as time passes. If the length of the edge is increasing at 1cm/sec, how fast is the volume changing when the edge length is 20cm?

$\frac{dV}{dt} = \frac{dL}{dt} \times \frac{dV}{dL}$

Volume $= L^3$

$\frac{dV}{dL} = 3L^2$

$\frac{dL}{dt} = 1$

$\frac{dV}{dt} = 1 \times 3L^2 = 3 \times 20^2 = \underline{\underline{1200\,cm^3/sec}}$.

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4.

A spherical balloon is being inflated. When the radius is 10 cm the surface area is increasing at a rate of $120\pi\,cm^2\,s^{-1}$.

Find the rate at which the volume is increasing at this moment. **5**

(Volume of sphere $= \frac{4}{3}\pi r^3$, surface area $= 4\pi r^2$)

$\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dA} \times \frac{dA}{dt}$

$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dr} = 4\pi r^2$

$A = 4\pi r^2$

$\frac{dA}{dr} = 8\pi r$

$\frac{dr}{dA} = \frac{1}{8\pi r}$

$\frac{dA}{dt} = 120\pi$

$\frac{dV}{dt} = 4\pi r^2 \times \frac{1}{8\pi r} \times 120\pi$

$= 60\pi r\ cm^3/sec$

$r = 10 \Rightarrow \underline{\underline{600\pi\,cm^3 s^{-1}}}$

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5.

The height of a cube is increasing at the rate of $5\,cm\,s^{-1}$.

Find the rate of increase of the volume when the height of the cube is 3 cm. **4**

$\frac{dV}{dt} = \frac{dh}{dt} \times \frac{dV}{dh}$

$V = h^3$

$\frac{dV}{dh} = 3h^2$

$\frac{dh}{dt} = 5$

$\frac{dV}{dt} = 5 \times 3h^2 = 15h^2$

at $h = 3 \Rightarrow 15 \times 9 = \underline{\underline{135\,m^3 s^{-1}}}$