Addition Formulae Homework Solutions

Total = 4[illegible]1

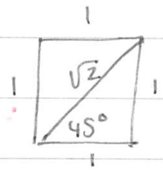
Q1. (a) $\sin(x+90)^{\circ}$

$= \sin x \cos 90^{\circ} + \cos x \sin 90^{\circ}$ ✓

$= 0 + \cos x = \cos x$ ✓ 2/2

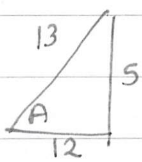
(b) $\sin 135^{\circ} = \sin(45^{\circ}+90^{\circ})$ ✓

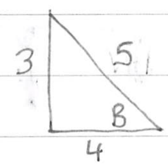
(from previous answer) $= \cos 45^{\circ}$ ✓ $= \dfrac{1}{\sqrt{2}}$ ✓ 3/3



Q2. $\sin A = \frac{5}{13}$ $\quad \cos B = \frac{4}{5}$ $\quad \sin(A+B)$

$\sin(A+B) = \sin A \cos B + \cos A \sin B$ ✓





$\left(\frac{5}{13}\right)\left(\frac{4}{5}\right)$ ✓ $+ \left(\frac{12}{13}\right)\left(\frac{3}{5}\right)$ ✓

$= \frac{20}{65} + \frac{36}{65} = \frac{56}{65}$ ✓ 4/4

Q3. Prove $\cos(A+B)\cos B + \sin(A+B)\sin B = \cos A$

LHS

$(\cos A \cos B - \sin A \sin B)\cos B + (\sin A \cos B + \cos A \sin B)\sin B$ ✓

$= \cos A \cos^2 B - \cancel{\sin A \sin B \cos B} + \cancel{\sin A \sin B \cos B} + \cos A \sin^2 B$ ✓

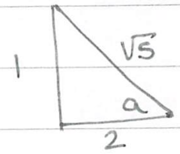
$= \cos A \cos^2 B + \cos A \sin^2 B$ ✓

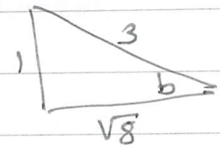
$= \cos A(\cos^2 B + \sin^2 B)$

$= \cos A\,(1)$ ✓

$= \cos A = \text{RHS}$ ✓ 5/5

Q4. Show $\sin(a-b) = \dfrac{2\sqrt{2}-2}{3\sqrt{5}}$





LHS

$\sin(a-b) = \sin a\cos b - \cos a\sin b$ ✓

$= \left(\tfrac{1}{\sqrt{5}}\right)\left(\tfrac{\sqrt{8}}{3}\right) - \left(\tfrac{2}{\sqrt{5}}\right)\left(\tfrac{1}{3}\right)$ ✓ ✓

$= \sqrt{8}/3\sqrt{5} - \dfrac{2}{3\sqrt{5}}$ ✓

$= \dfrac{\sqrt{8}-2}{3\sqrt{5}} = \dfrac{2\sqrt{2}-2}{3\sqrt{5}} = \text{RHS}$ ✓

5/5

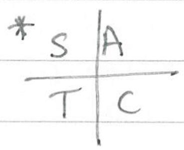
Q5. $\cos 80^\circ \cos 40^\circ - \sin 80^\circ \sin 40^\circ$

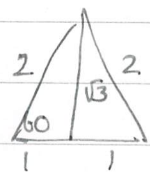
$= \cos(80+40)^\circ$ ✓

$= \cos 120^\circ$ ✓

$= -\cos 60^\circ$ ✓

$= -\tfrac{1}{2}$ ✓ 4/4





Q6. $\cos 2x^\circ - 3\cos x^\circ + 2 = 0$

$= (\cos^2 x^\circ - \sin^2 x^\circ) - 3\cos x^\circ + 2 = 0$ ✓

$= \cos^2 x^\circ - (1 - \cos^2 x^\circ) - 3\cos x^\circ + 2 = 0$

$= 2\cos^2 x^\circ - 1 - 3\cos x^\circ + 2 = 0$ ✓

$= 2\cos^2 x^\circ - 3\cos x^\circ + 1 = 0$

$(2\cos x^\circ - 1)(\cos x^\circ - 1) = 0$ ✓

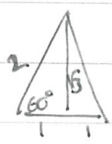
$2\cos x^\circ = 1$ ✓ $\qquad \cos x^\circ = 1$

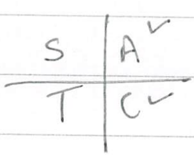
$\cos x^\circ = \tfrac{1}{2}$ $\qquad x = 0^\circ$ ✓, 360°

$x = 60^\circ$

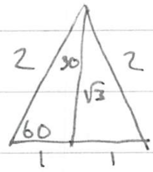
$x = 300^\circ$ ✓

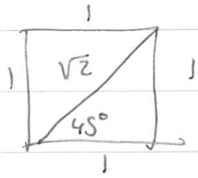
5/5





Q7. (a) $\frac{7\pi}{12} = \frac{\pi}{3} + \frac{\pi}{4}$ $\quad \sin\left(\frac{7\pi}{12}\right)$





$= \sin\left(\frac{\pi}{3} + \frac{\pi}{4}\right) = \sin\pi/3 \cos\pi/4 + \cos\pi/3 \sin\pi/4$

$= \left(\sqrt{3}/2\right)\left(1/\sqrt{2}\right) + \left(1/2\right)\left(1/\sqrt{2}\right)$

$= \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}} = \frac{\sqrt{3}+1}{2\sqrt{2}} \frac{\times\sqrt{2}}{\times\sqrt{2}} = \frac{\sqrt{6}+\sqrt{2}}{4}$

5/5

(b) Show $\sin(A+B) + \sin(A-B) = 2\sin A\cos B$

$= \sin A\cos B + \cos A\sin B + \sin A\cos B - \cos A\sin B$

$= 2\sin A\cos B = RHS$

3/3

(c) (i) $\pi/12 = \frac{\pi}{3} - \frac{\pi}{4}$ 1/1

$\frac{\pi}{3} = \frac{4\pi}{12} \quad \frac{\pi}{4} = \frac{3\pi}{12}$

(ii) $\sin\left(\frac{7\pi}{12}\right) + \sin\left(\frac{\pi}{12}\right)$

$\sin\left(\frac{\pi}{12}\right) = \sin\left(\frac{\pi}{3} - \frac{\pi}{4}\right) \quad \sin\left(\frac{\pi}{3} + \frac{\pi}{4}\right) + \sin\left(\frac{\pi}{3} - \frac{\pi}{4}\right)$

See Previous Answer

$2\sin A\cos B$

$= 2\sin\pi/3 \cos\pi/4$

$= 2\left(\sqrt{3}/2\right)\left(1/\sqrt{2}\right) = \frac{2\sqrt{3}}{2\sqrt{2}} \frac{\times\sqrt{2}}{\times\sqrt{2}} = \frac{2\sqrt{6}}{4} = \frac{\sqrt{6}}{2}$

4/4