Higher Derivatives

The first derivative is represented by $f'(x)$ or dy/dx. The second is represented by $f''(x)$ or $\frac{d^2y}{dx^2}$. The third is $f'''(x)$ or $f^3(x)$ or $\frac{d^3y}{dx^3}$ and so on.

How we use higher derivatives

1. Finding Stationary Points

$\frac{d^2y}{dx^2}$ finds the nature of the S.P's. (Rate of change of the gradient).

Recall that $\frac{d^2y}{dx^2} > 0 \Rightarrow \min .S.P.$

$\frac{d^2y}{dx^2} < 0 \Rightarrow \max .S.P.$

$\frac{d^2y}{dx^2} = 0 \Rightarrow$ Use a nature table

Stationary Points

Example: Find the S.P's for $y = 2x^3 - 3x^2 - 12x$ and determine their nature.

at S.P's $dy/dx = 0$

$dy/dx = 6x^2 - 6x - 12 = 0$

$x^2 - x - 2 = 0$

$(x - 2)(x + 1) = 0$

$x = 2$, $x = -1$

$y = -20$, $y = 7$

$\underline{(2,-20)}$ $\underline{(-1,7)}$

$d^2y/dx^2 = 12x - 6$

at $x = 2$ $12(2) - 6 = 18 > 0$

$x = -1$ $12(-1) - 6 = -18 < 0$

min. S.P. at $(2,-20)$

max. S.P. at $(-1,7)$

How we use higher derivatives

2. Displacement, velocity and acceleration

Recall, velocity is the rate of change of displacement with respect to time.

$$v = \frac{ds}{dt} \quad \text{or } s'(t)$$

Acceleration is the rate of change of velocity with respect to time.

$$a = \frac{dv}{dt} = \frac{d^2s}{dt^2} \quad \text{or } s''(t)$$

If $\frac{d^2s}{dt^2} > 0 \Rightarrow$ acceleration, if $\frac{d^2s}{dt^2} < 0 \Rightarrow$ deceleration.

Displacement, velocity and acceleration

Examples:

1. A pebble is thrown upwards from a point A. The height of the pebble, s metres above A after t seconds is given by the formula

$$s = 8t - 2t^2$$

(a) Calculate the velocity of the pebble at the point at which it is thrown.

$s'(t) = 8 - 4t$ $t=0$

$s'(0) = \underline{8\text{ m/s}}$

(b) Calculate the acceleration of the pebble as it lands and after how many seconds does it take to land?

$s''(t) = -4\text{ m/s}^2$

decelerating at 4 m/s^2

$8t - 2t^2 = 0$

$2t(4 - t) = 0$

$t = 0$ and $t = 4$

4 seconds to land

Displacement, velocity and acceleration

Examples:

2. The displacement s metres, of point P at time t seconds is given by the formula $s = 4t^3 - t^2 + 20t - 10$

(a) Find the velocity of P after 5 seconds

$s'(t) = 12t^2 - 2t + 20$ $s'(5) = 12(5)^2 - 2(5) + 20 = \underline{310\text{ m/s}}$

(b) Find the time at which the acceleration of P is 22 m/s^{-2}

$s''(t) = 24t - 2$

$24t - 2 = 22$

$24t = 24$

$\underline{t = 1}$ second