Daily Practice **8.9.2015**

Q1. State the centre and radius of the circle $x^2 + y^2 - 4x + 2y - 20 = 0$

Centre = $(2, -1)$ $r = \sqrt{2^2+1^2+20} = \sqrt{25} = \underline{\underline{5}}$

Q2. The length of the radius of the circle $x^2 + y^2 - 8x + 10y + k = 0$ is 7. Find the value of k

$r = \sqrt{g^2+f^2-c} \Rightarrow 7 = \sqrt{4^2+5^2-k}$ $7 = \sqrt{41-k}$

$49 = 41 - k$ $\underline{\underline{k = -8}}$

Q3. Does the recurrence relation $u_{n+1} = 0.75u_n + 3$ have a limit? If so, find the value of the limit

$-1 < 0.75 < 1 \Rightarrow$ a limit exists

$L = \frac{b}{1-a} \Rightarrow \frac{3}{1-0.75} = \underline{\underline{12}}$

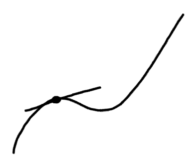
Today we will be learning how to differentiate.

Homework Due Tomorrow

Functions

A function is either written y or f(x)

The derivative of a function is written f'(x) or dy/dx



Differential Calculus

Calculus is the mathematical study of change. Problems involving motion are solved using calculus.

* Differentiation is the instantaneous rate of change of a function. *

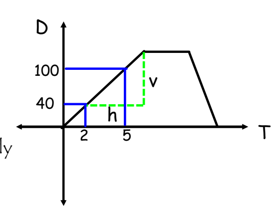
Given an S, D, T graph,

the speed remains constant

for each section of the journey.

To calculate the speed, you can simply

find the gradient of the graph

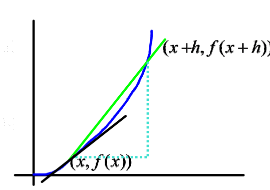


Differential Calculus

But realistically, the speed of a car is continuously changing and therefore is more likely to have a curved graph

$$\frac{f(x+h) - f(x)}{x - x + h}$$

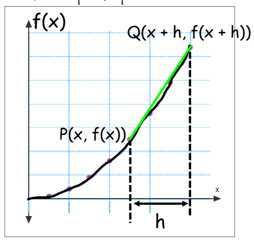
$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$



Differential Calculus

Calculating the gradient this way is called "from first principles"

$m_{pq} =$



Differential Calculus

Example: Differentiate x^2 from first principles

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Let $f(x) = x^2$

$f(x+h) = (x+h)^2$

$$\Rightarrow \frac{(x+h)^2 - x^2}{h} = \frac{\cancel{x^2} + 2xh + h^2 - \cancel{x^2}}{h}$$

$$= \frac{\cancel{h}(2x+h)}{\cancel{h}} = 2x + h \quad \lim_{h \to 0} 2x + h = \underline{\underline{2x}}$$

Differential Calculus 8.9.15

Differentiation means calculating the instantaneous rate of change of any function. The <u>derivative represents the gradient of the tangent</u> to the graph of the function.

Notation: f(x) = the function f'(x) = the derivative of the function

Leibniz Notation: y = the function dy/dx = the derivative of the function

$f(x)$	$f'(x)$
x^2	$2x$
x	1 or x^0
$3x^3$	$9x^2$
$4x$	4
$3x-1$	3
$2x^2+1$	$4x$
x^4	$4x^3$
x^5	$5x^4$

Differential Calculus

a is a constant n is an element of Rational numbers

$$\boxed{\text{If } f(x) = ax^n \text{ where } n \in \mathbb{Q} \text{ then } f'(x) = anx^{n-1}}$$

Examples: Differentiate the following

1) $f(x) = x^4$ 2) $f(x) = 3x^2$ 3) $f(x) = 5x^{-3}$

$f'(x) = 4x^3$ $f'(x) = 6x$ $f'(x) = -15x^{-4}$

$= \dfrac{-15}{x^4}$

Ex 6D, 6F

Q1-10 Q1-10

Daily Practice 9.9.2014

Q1. Find the equation of the perpendicular bisector of (3, 1) and (1, 5)

$m = \dfrac{5-1}{1-3} = \dfrac{4}{-2} = -2$ $\perp m = \frac{1}{2}$ Midpt $= \left(\frac{3+1}{2}, \frac{1+5}{2}\right) = (2, 3)$

$y - 3 = \frac{1}{2}(x - 2)$

Q2. If the line 2x - 3y + 5 = 0 is perpendicular to 3x + ky - 8 = 0, find the value of k

$3y = 2x + 5$

$y = \frac{2}{3}x + \frac{5}{3}$

$m = \frac{2}{3}$ $\perp m = \frac{-3}{2}$

$ky = -3x + 8$

$y = \frac{-3}{k}x + 8$

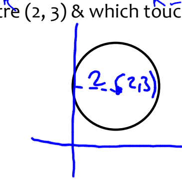
$\frac{-3}{k} = \frac{-3}{2}$

$k = 2$

Q3. Find the equation of the circle with centre (2, 3) & which touches the y - axis

$(x-a)^2 + (y-b)^2 = r^2$

$(x-2)^2 + (y-3)^2 = 2^2$



Today we will be continuing to practise differentiation.

Homework Online due 15.9.15

Differential Calculus

Remember!! Before differentiating, you must ensure that your function is in the form x^n so get rid of roots by using fractional indices.

$$(\sqrt[y]{a})^x = a^{\frac{x}{y}} \quad \text{and} \quad \frac{1}{x^m} = x^{-m} \quad ax^{-m} = \frac{a}{x^m}$$

1) $f(x) = \frac{1}{\sqrt{x}}$

$= \frac{1}{x^{\frac{1}{2}}} = x^{-\frac{1}{2}}$

$f'(x) = -\frac{1}{2}x^{-3/2}$

$-\frac{1}{2}\frac{1}{x^{3/2}}$

$-\frac{1}{2x^{3/2}} = -\frac{1}{(\sqrt{x})^3}$

2) $f(x) = 2x^{\frac{3}{4}}$

$f'(x) = \frac{6}{4}x^{-\frac{1}{4}}$

$= \frac{6}{4x^{\frac{1}{4}}}$

$= \frac{6}{4\sqrt[4]{x}}$

3) $f(x) = \frac{2}{\sqrt[3]{x^2}} = \frac{2}{x^{\frac{2}{3}}}$

$f(x) = 2x^{-\frac{2}{3}}$

$f'(x) = -\frac{4}{3}x^{-\frac{5}{3}}$

$= \frac{-4}{3x^{\frac{5}{3}}}$

$= \frac{-4}{3\sqrt[3]{x^5}}$

Differential Calculus 9.9.15

$\boxed{\begin{array}{l}\text{If } f(x) = ax \text{ then } f'(x) = a \\ \text{If } f(x) = a \text{ then } f'(x) = 0\end{array}}$

'a' is known as a constant (a number with a fixed value)

Examples: Differentiate with respect to x

(i) $f(x) = 3x^1$ (ii) $f(x) = 2$ (iii) $f(x) = 4x^8$

$f'(x) = 3$ $f'(x) = 0$ $f'(x) = 32x^7$

$6x^0$ $3x^1$

$0x^{-1}$ $3x^0$

$= \underline{0}$ $= \underline{3}$

$x^0 = 1$

Differentiating sums/differences

If $f(x) = g(x) \pm h(x)$ $\quad f'(x) = g'(x) \pm h'(x)$

Differentiate each term seperately using the usual rules

Examples: Differentiate the following with respect to x

i) $f(x) = x^3 + 3x^2 + 2x - 7$

$f'(x) = 3x^2 + 6x + 2$

ii) $f(x) = \frac{1}{2}x^2 - \frac{3}{\sqrt[3]{x^2}}$

$f(x) = \frac{1}{2}x^2 - \frac{3}{x^{\frac{2}{3}}}$

$= \frac{1}{2}x^2 - 3x^{-\frac{2}{3}}$

$= 1x + 2x^{-5/3}$

$= x + \frac{2}{x^{5/3}}$

$= x + \frac{2}{(\sqrt[3]{x})^5}$

Daily Practice 10.9.15

Q1. State the size of the angle the line y - 2x = 4 makes with the positive direction of the x - axis

$y - 2x = 4$

$y = 2x + 4 \quad m = 2$

$m = \tan\theta$

$2 = \tan\theta$

$\tan^{-1}(2) = \underline{63.4°}$

Q2. Find the equation of the line that is perpendicular to y - 2x + 1 = 0 and passes through (3, 1)

$y = 2x - 1$

$m = 2$

$\perp m = -\frac{1}{2}$

$y - 1 = -\frac{1}{2}(x - 3)$ ×2

$2y - 2 = -1x + 3$

$2y + x - 5 = 0$

$y = -\frac{1}{2}x + \frac{5}{2}$

Q3. Write in completed square form $5x^2 + 30x - 8$

$5\left[x^2 + 6x - \frac{8}{5}\right] = 5\left[(x+3)^2 - 9 - \frac{8}{5}\right]$

$= 5(x+3)^2 - 53$

Q4. The number of houses in a particular area increases by 4% per year. The number of homes demolished in the same area each year is 3. Write a recurrence relation to show the number of homes in n + 1 years

$U_{n+1} = 1.04U_n - 3$

Today we will be learning to differentiate products and quotients.

Homework online due 15.9.15

Target Setting

Personal Support Journal Page 40.

Differentiate with respect to x

1. $f(x) = 3x^2 + 2$ $\quad f'(x) = 6x$
2. $f(x) = 4x^3 - 5x^2 + 12x - 3$
   $f'(x) = 12x^2 - 10x + 12$
3. $f(x) = x^{\frac{2}{3}} - 4x$
   $f'(x) = \frac{2}{3}x^{-\frac{1}{3}} - 4 = \frac{2}{3\sqrt[3]{x}} - 4$
4. $f(x) = \frac{3}{\sqrt[3]{x}} + 4x^5 = 3x^{-\frac{1}{3}} + 4x^5$
   $f'(x) = -x^{-\frac{4}{3}} + 20x^4$
   $= -\frac{1}{\sqrt[3]{x}^4} + 20x^4$
5. $f(x) = 9x^2 + x^{-\frac{1}{2}}$
   $f'(x) = 18x - \frac{1}{2}x^{-\frac{3}{2}}$
   $= 18x - \frac{1}{2\sqrt{x}^3}$

Extra questions if finished: Differentiate

$f(x) = 3x^2 + 2x - 4$

$y = 5x^2 + 4$

$f(x) = \frac{1}{\sqrt[3]{x^2}} + 5x - 2$

$f(x) = 4\sqrt{x} + 7x^2 + 9$

Rules of Indices: $a^m \times a^n = a^{m+n}$

$(a^m)^n = a^{mn}$

$a^m \div a^n = a^{m-n}$

$a^0 = 1$

$a^{-1} = \frac{1}{a}$ and $a^{-m} = \frac{1}{a^m}$

$a^{\frac{1}{2}} = \sqrt{a}$ and $a^{\frac{1}{q}} = \sqrt[q]{a}$

$a^{\frac{p}{q}} = (a^p)^{\frac{1}{q}} = \sqrt[q]{a^p}$

$a^{\frac{p}{q}} = (a^{\frac{1}{q}})^p = (\sqrt[q]{a})^p$

Ex. 6F

Continue working through Q's

Daily Practice 11.9.2015

Q1. Solve the equation $x^2 - x - 6 = 0$

$(x+2)(x-3) = 0$

$x + 2 = 0 \quad \underline{x = -2}$ $\qquad x - 3 = 0 \quad \underline{x = 3}$

Q2. Find the value of t if the lines $2x + ty + 5 = 0$ and $(t + 6)x + 2y - 9 = 0$ are perpendicular

$ty = -2x - 5$

$y = -\frac{2}{t}x - 5$

$2y = -(t+6)x + 9$

$y = -\frac{(t+6)}{2}x + 9$

$m_1 = \frac{-2}{t} \quad m_2 = \frac{-(t+6)}{2}$

$m_1 \times m_2 = -1$

$\frac{-2}{t} \times \frac{-t-6}{2} = -1$

$\frac{2t+12}{2t} = -1$

$2t + 12 = -2t$

$12 = -4t$

$\underline{\underline{t = -3}}$

Q3. Find $f'(x)$ when $f(x) = 3x^{-2} + \frac{1}{2\sqrt{x}}$

$f(x) = 3x^{-2} + \frac{1}{2}x^{-\frac{1}{2}}$

$f'(x) = -6x^{-3} - \frac{1}{4}x^{-\frac{3}{2}}$

$= -\frac{6}{x^3} - \frac{1}{4\sqrt{x}^3}$

Today we will be learning to differentiate products and quotients.

Homework online due 15.9.15

Differentiating products (x) and quotients (÷) 11.9.15

Express as a sum of individual terms first by:

- Multiplying out brackets where possible
- Expressing surds as fractional indices
- Split any fractions into individual fractions to get seperate terms

Differentiating products (x) and quotients (÷)

Examples: Differentiate

1. $y = (2x + 5)(x - 4) = 2x^2 - 8x + 5x - 20$

$y = 2x^2 - 3x - 20$

$dy/dx = 4x - 3$

2. $f(x) = \dfrac{5-2x}{x^2}$

$f(x) = x^{-2}(5 - 2x^1)$

$f(x) = 5x^{-2} - 2x^{-1}$

$f'(x) = -10x^{-3} + 2x^{-2}$

$f'(x) = -\dfrac{10}{x^3} + \dfrac{2}{x^2}$

3. $f(x) = x^3(3 - \dfrac{2}{x^4})$

$f(x) = 3x^3 - \dfrac{2x^3}{x^4}$

$f(x) = 3x^3 - 2x^{-1}$

$f'(x) = 9x^2 + 2x^{-2}$

$f'(x) = 9x^2 + \dfrac{2}{x^2}$

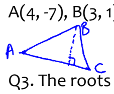
Daily Practice 14.9.15

Q1. State the gradient of the line 3y - x + 2 = 0

$3y = x - 2$

$y = \frac{1}{3}x - \frac{2}{3}$ $m = \frac{1}{3}$

Q2. State the equation of the altitude from B in the triangle A(4, -7), B(3, 1) and C(5, -6)



$\perp m = -1$ $y - b = m(x - a)$

$m_{AC} = \dfrac{-6+7}{5-4} = 1$ $y - 1 = -1(x - 3)$

Q3. The roots of (x - 1)(x + k) = -4 are equal, find the values of k

$b^2 - 4ac = 0$ $x^2 + kx - x - k = -4$ $c = -k + 4$

$a = 1$ $b = k - 1$ (1) $x^2 + \underbrace{kx - x}_{x(k-1)} - k + 4 = 0$ (2)

Q4. The straight line y = x cuts the circle $x^2 + y^2 - 6x - 2y - 24 = 0$ at A and B. Find the coordinates of A and B

Sub. (1) into (2)

$x^2 + x^2 - 6x - 2x - 24 = 0$

$2x^2 - 8x - 24 = 0$

$x^2 - 4x - 12 = 0$

$(x + 2)(x - 6)$

$x = 6$ $x = -2$

$y = 6$ $x = -2$

B(6, 6) A(-2, -2)

Q3. The roots of (x - 1)(x + k) = -4 are equal, find the values of k

$b^2 - 4ac = 0$ $x^2 + kx - x - k = -4$ $c = -k + 4$

$a = 1$ $b = k - 1$ (1) $x^2 + \underbrace{kx - x}_{x(k-1)} - k + 4 = 0$ (2)

$b^2 - 4ac = 0$

$\underline{(k-1)^2 - 4(1)(-k+4)} = 0$

$k^2 - k - k + 1 + 4k - 16 = 0$

$k^2 - 2k - 15 + 4k = 0$

$k^2 + 2k - 15 = 0$

$(k + 5)(k - 3) = 0$

$\underline{k = -5}$, $\underline{k = 3}$

$(x + 3)(x + 2)$

Today we will be continuing to practise differentiation of quotients and products.

Homework Due Tomorrow!

$(\sqrt{x} + 5)(x^2 - 1)$

$\dfrac{x^{\frac{1}{3}} + 2}{\sqrt[3]{x^2}}$

Differentiating products (x) and quotients (÷)

Examples:

3. $f(x) = \dfrac{(1 - \sqrt[3]{x})^2}{x^{\frac{1}{3}}}$ = $f(x) = x^{-\frac{1}{3}}(1 - x^{\frac{1}{3}})^2$

$f(x) = x^{-\frac{1}{3}}(1 - 2x^{\frac{1}{3}} + x^{\frac{2}{3}}) = x^{-\frac{1}{3}} - 2x^0 + x^{\frac{1}{3}}$

$= x^{-\frac{1}{3}} - 2 + x^{\frac{1}{3}}$

$f'(x) = -\frac{1}{3}x^{-\frac{4}{3}} + \frac{1}{3}x^{-\frac{2}{3}}$

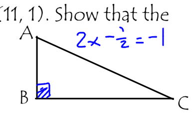
* $f'(x) = -\dfrac{1}{3\sqrt[3]{x^4}} + \dfrac{1}{3\sqrt[3]{x^2}}$

$$f(x)=\frac{(\sqrt{x}+\sqrt[4]{x})^2}{\sqrt{x^3}}$$

http://missdeely.weebly.com/s56-higher.html

Daily Practice 15.9.15

Q1. A triangle has vertices A(1, 1) B(3, 5) and C(11, 1). Show that the triangle ABC is right angled at B.



$2 \times -\frac{1}{2} = -1$

$m_{AB} = \frac{5-1}{3-1} = \frac{4}{2} = 2$ $m_{BC} = \frac{1-5}{11-3} = -\frac{1}{2}$

Since $m_1 \times m_2 = -1$

$\Rightarrow$ Right angle $A\hat{B}C$

Q2. A recurrence relation is defined as $u_{n+1} = au_n + 4$, $u_0 = 2$.

Given that the limit to the above relation is 10, find a.

$L = \frac{b}{1-a}$ $\quad 10 = \frac{4}{1-a}$ $\quad \times 1-a \quad \times 1-a$ $\quad 10 - 10a = 4$ $\quad -10a = -6$ $\quad a = \frac{6}{10}$ or 0.6 or $\frac{3}{5}$

Q3. Differentiate the function $y = 4x^3 + 3x^2 + 2x + 1$

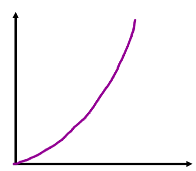
$dy/dx = 12x^2 + 6x + 2$

Today we will be working out rate of change questions.

Homework due!

Rates of change 15.9.15

Remember that the derivative of a function represents the rate at which the y - coordinate changes with respect to the x - coordinate.



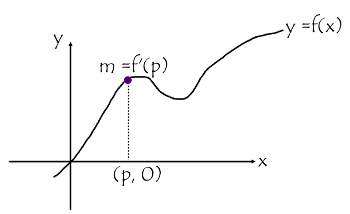
In a SDT graph, the rate of change is the speed.

In a graph with height and time, the rate of change is velocity

Evaluating derivatives

The rate of change of a function can be evaulated at any point along the curve by substituting in a value for x .

$f'(p)$ means finding the value of the derivative at the point where $x = p$



Evaluating derivatives

Examples:

1. If $f(x) = (3x - 5)^2$, find the value of $f'(3)$

$f(x) = 9x^2 - 15x - 15x + 25 = 9x^2 - 30x + 25$

$f'(x) = 18x - 30 \qquad f'(3) = 18(3) - 30 = \underline{24}$

2. If $f(x) = 4x^{2/3} + 5x - 2$, find the value of $f'(x)$ when $x = 8$

$f'(x) = \frac{8}{3}x^{-\frac{1}{3}} + 5 = \frac{8}{3x^{\frac{1}{3}}} + 5 = \frac{8}{3\sqrt[3]{x}} + 5$

when $x = 8$

$f'(8) = \frac{8}{3\sqrt[3]{8}} + 5 = \frac{4}{3} + 5 = \frac{19}{3}$

Daily Practice 16.9.15

Q1. Differentiate $\dfrac{2}{\sqrt[3]{x^4}}$ with respect to x

$2x^{-\frac{4}{3}}$ $f'(x) = -\frac{8}{3}x^{-\frac{7}{3}} = \dfrac{-8}{3\sqrt[3]{x}^{7}}$

Q2. State the equation of the line parallel to 6x - 2y + 3 = 0 that contains the point (1, 4)

$2y = 6x + 3$
$y = 3x + \frac{3}{2}$
$m = 3$

$y - b = m(x - a)$
$y - 4 = 3(x - 1)$

Q3. Calculate the limit of the recurrence relation $u_{n+1} = 0.4u_n + 36$

$L = \frac{b}{1-a} = \frac{36}{1-0.4} = \frac{36}{0.6} = 60$

Today we will be continuing to practise differentiation questions in context.

Homework Online due 22.9.15

1. $f(x) = 3x^3 - 4x$. Calculate the value of $f'(1)$. 5

2. $f(x) = (2x - 1)^2$. Find $f'(-2)$ -20

3. $y = 4x^2 - 3x + 5$. Calculate the value of $\frac{dy}{dx}$ when $x = 2$.

$dy/dx = 8x - 3 = 8(2) - 3 = 13$

4. $y = \dfrac{x^2 - 1}{x}$. Find the value of $\frac{dy}{dx}$ when $x = 3$.

$x^{-1}(x^2 - 1)$
$x^1 - x^{-1}$
$dy/dx = 1 + x^{-2}$
$= 1 + \frac{1}{x^2}$
$= 1 + \frac{1}{3^2}$
$= 1\frac{1}{9}$ or $\frac{10}{9}$

5. $f(x) = \sqrt{x}(4 + 2\sqrt{x})$. Find $f'(4)$.

6. $f(x) = x^3(x - 1)$. Find the value of $f'(-1)$.

7. $y = \dfrac{x - 3x^2}{x^3}$. Calculate the value of $\frac{dy}{dx}$ when $x = -2$.

8. $f(x) = \left(x + \frac{1}{x}\right)^2$. Find $f'(½)$.

9. $f(x) = \dfrac{x^2 - 2x}{\sqrt{x}}$. Calculate $f'(16)$.

Example in context: 16-9.15

A pebble is thrown vertically upwards from a point A. The height of the pebble, s metres, above A after t seconds is given by the formula:

$$s(t) = 8t - 2t^2$$

(a) Calculate the velocity of the pebble at the point at which it is thrown.

$s'(t) = 8 - 4t$ $t = 0$ $s'(0) = 8 - 4(0) = 8\,m/s$

(b) Calculate the velocity of the pebble after 3 seconds.

$s'(3) = 8 - 4(3) = -4\,m/s$

Q1. The distance, d metres, travelled on a fairground ride is calculated using the formula $d(t) = 8t^2 - 4t$, where t is the time in seconds after the start of the ride. Calculate the speed of the ride after 3 seconds.

Ex. 6H Q5 - 14

$d'(t) = 16t - 4$
$d'(3) = 16(3) - 4 = 44\,m/s$

Q2. The distance a rocket travels is calculated using the formula $d(t) = 4t^3$, where t is the time in seconds after lift-off. Calculate the speed of the rocket after 8 seconds.

$d'(t) = 12t^2$
$d'(8) = 12(8)^2 = 12 \times 64 = 768\,/s$

$C(t) = 2\pi\sqrt{t}$
$C(t) = 2\pi t^{\frac{1}{2}}$
$C'(t) = \pi t^{-\frac{1}{2}}$
$C'(t) = \frac{\pi}{t^{1/2}}$
$= \frac{\pi}{\sqrt{t}}$

Daily Practice 17.9.15

Q1. $y = \dfrac{x - 3x^2}{x^3}$. Calculate the value of $\frac{dy}{dx}$ when $x = -2$

$x^{-3}(x - 3x^2) = x^{-2} - 3x^{-1}$ $dy/dx = -2x^{-3} + 3x^{-2}$ $\frac{-2}{x^3} + \frac{3}{x^2}$ $x = -2$ $\frac{-2}{(-2)^3} + \frac{3}{(-2)^2}$

$\frac{-2}{-8} + \frac{3}{4} = \frac{1}{4} + \frac{3}{4} = 1$

Q2. State the size of the obtuse angle that the line y - 3x = 4 makes with the x-axis

$m = 3$ $y = 3x + 4$
$m = \tan\theta$
$3 = \tan\theta$
$\theta = \tan^{-1}(3)$
$\theta = 71.56$
$180° - 71.56° = 108.4°$

Q3. Given the recurrence relation $u_{n+1} = ku_n + 5$, find the value of k for which the limit is 4.

$L = \frac{b}{1-a} =$

$4 = \frac{5}{1-k}$
$\times(1-k)$ $\times(1-k)$
$4 - 4k = 5$
$-4k = 1$
$k = -\frac{1}{4}$ or -0.25

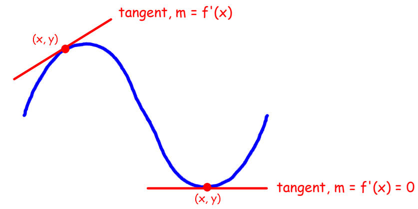
Today we will be learning about the gradient of the tangent and equation of tangent.

Homework due 22.9.15

Gradient & Equation of the Tangent 17.9.15

The gradient of the tangent to the curve is the same as the gradient of the curve at a specific point.

Gradient tangent (m) = derivative of curve f'(x)



Gradient & Equation of the Tangent

Examples

1) Find the gradient of the tangent to each curve at the given value for x.

a) $f(x) = 16 - 3x^2$ at $x = -2$

$f'(x) = -6x$

$f'(-2) = -6(-2) = \underline{\underline{12}} = m$

Gradient & Equation of the Tangent

b) $f(x) = \dfrac{5 - 2x}{x^2}$ at $(-2, -5)$

$f(x) = x^{-2}(5-2x) = 5x^{-2} - 2x^{-1}$

$f'(x) = -10x^{-3} + 2x^{-2}$

$f'(x) = \dfrac{-10}{x^3} + \dfrac{2}{x^2}$

$x = -2$

$f'(-2) = \dfrac{-10}{(-2)^3} + \dfrac{2}{(-2)^2} = \dfrac{-10}{-8} + \dfrac{2}{4} = 1\frac{1}{4} + \frac{2}{4} = \underline{\underline{1\frac{3}{4}}} = m$

Gradient & Equation of the Tangent

To find the equation of the tangent $y - b = m(x - a)$

Differentiate the curve to get the gradient, sub. in value for x and then use the point of contact to create the equation.

Gradient & Equation of the Tangent

Examples

1) Find the equation of the tangent to the curve $y = x^2 - 2x + 2$ at the point $(\underline{-3}, 17)$.

Gradient & Equation of the Tangent

2) Find the equation of the tangent to the curve $y = \sqrt{x^3}$ at $x = 4$.

* First finding y

$y = \sqrt{4^3} = 8$

$(4,8)$ = point of contact

$m = \frac{dy}{dx}$ $\quad y = x^{\frac{3}{2}}$

$\frac{dy}{dx} = \frac{3}{2}x^{\frac{1}{2}} = \frac{3\sqrt{x}}{2}$

at $x = 4$ $\quad \frac{3\sqrt{4}}{2} = \frac{6}{2} = 3 = m$

$y - 8 = 3(x - 4)$

$y - 8 = 3x - 12$

$3x - y - 4 = 0$

Ex. 61

Q4 (b)

Ex. 6J Q1 b, f, h

Q2, 4, 6, 8

Daily Practice 18.9.15

Q1. Differentiate $\frac{3x+1}{x^2\sqrt{x}} = \frac{3x+1}{x^{\frac{5}{2}}} = \frac{3x}{x^{\frac{5}{2}}} + \frac{1}{x^{\frac{5}{2}}} = 3x^{-\frac{3}{2}} + x^{-\frac{5}{2}}$

$f'(x) = -\frac{9}{2}x^{-\frac{5}{2}} - \frac{5}{2}x^{-\frac{7}{2}} = -\frac{9}{2\sqrt{x^5}} - \frac{5}{2\sqrt{x^7}}$

Q2. (a) State the centre and the radius of the circle $x^2 + y^2 + 18x + 20y + 81 = 0$ ②

(b) State the point(s) of intersection of the line $y = x + 1$ ① and the above circle

(a) Centre $= (-9, -10)$ Radius $= \sqrt{9^2 + 10^2 - 81} = \sqrt{100} = 10$

(b) Sub. ① into ②

$x^2 + (x+1)^2 + 18x + 20(x+1) + 81 = 0$

$x^2 + x^2 + 2x + 1 + 18x + 20x + 20 + 81 = 0$

$2x^2 + 40x + 102 = 0$

$x^2 + 20x + 51 = 0$

$(x+3)(x+17) = 0$

$x = -3,\ x = -17$

$y = -3 + 1 \qquad y = -17 + 1$

$y = -2 \qquad y = -16$

$(-3, -2) \qquad (-17, -16)$

Find the equation of the tangent to the curve $y = 3x^3 - 4x + 1$ at the point $(1,0)$.

Find the equation of the tangent to the curve $y = \frac{4\sqrt{x}}{x} + 2x$ at the point where $x = 4$

$y = \frac{4x^{\frac{1}{2}}}{x} + 2x = 4x^{-\frac{1}{2}} + 2x$

A curve has equation $y = 3x^2 - 9x + 1$. A tangent to this curve has gradient 3. Find the equation of this tangent.

A curve has equation $y = \frac{x^4}{4} - 32x$. A tangent to this curve is parallel to the x-axis.

Find the equation of this tangent.

Daily Practice 22.9.15

Q1. Factorise $x^2 - 17x + 72$

$(x - 9)(x - 8)$

Q2. $f(x) = 3x^2 + 2x - 1$, find $f'(3)$

$f'(x) = 6x + 2 \qquad f'(3) = 6(3) + 2 = 20$

Q3. Simplify $3x^2(x^{1/2} + 2x^{-1})$

$3x^{\frac{5}{2}} + 6x = 3\sqrt{x^5} + 6x$

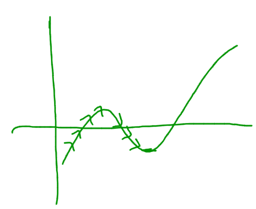
Q5. Show that the points P(4, 1), Q(5, 2) and R(7, 4) are collinear.

$m_{PQ} = \frac{2-1}{5-4} = \frac{1}{1} = 1 \qquad m_{QR} = \frac{4-2}{7-5} = \frac{2}{2} = 1$

- $m_{PQ} = m_{QR}$
- Share common point Q $\Rightarrow$ Collinear

Today we will be looking at increasing and decreasing functions.

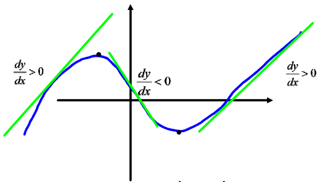
Homework due today.



Increasing and Decreasing Functions 22·9·15

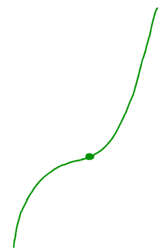
If the derivative of a curve is positive, this means that the function is increasing. $\frac{dy}{dx} > 0$

If the derivative of the curve is negative, it means that the function is decreasing. $\frac{dy}{dx} < 0$



To show that a curve is always increasing, you must show that dy/dx is always ≧ 0

To show that a curve is always decreasing, you must show that dy/dx is always ≦ 0



x^2 $-x^2$

Increasing and Decreasing Functions

Examples

1) For the function $f(x) = 3x^2 + 2x - 5$ state whether the function is increasing or decreasing at:

a) x = -3

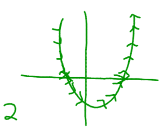
$f'(x) = 6x + 2$

at $x = -3$

$6(-3)+2 = -16 < 0$

$\Rightarrow$ decreasing

b) x = 4



$f'(4)$

$= 6(4)+2$

$= 26 > 0$

$\Rightarrow$ Increasing

Ex. 6L
pg. 106 Q1.

Increasing and Decreasing Functions

Examples:

2. Show that the function $y = x^3 + 2x$ is never decreasing

$dy/dx = 3x^2 + 2$

$3x^2 \geq 0$ always $\quad 2 \geq 0$ always

$3x^2 + 2 \geq 0 \Rightarrow$ Always Increasing

Increasing and Decreasing Functions

Examples

3) For $y = 2x^3 - x^2 - 5x - 9$ state whether the function is increasing or decreasing at x = -2.

Daily Practice 23.9.15

Q1. State the equation of the tangent to the curve $y = \frac{4\sqrt{x}}{x} + 2x$ at the point where x = 4

$y = \frac{4x^{\frac{1}{2}}}{x} + 2x = x^{-1}(4x^{\frac{1}{2}}) + 2x = 4x^{-\frac{1}{2}} + 2x$

$m = dy/dx = -2x^{-\frac{3}{2}} + 2$

$= -\frac{2}{x^{\frac{3}{2}}} + 2 = -\frac{2}{\sqrt{x^3}} + 2$

$x = 4$

$= \frac{-2}{\sqrt{4^3}} + 2 = -\frac{1}{4} + 2 = \frac{7}{4} = m$

$y = \frac{4\sqrt{x}}{x} + 2x$

$x=4 \quad y = \frac{4\sqrt{4}}{4} + 2(4) = 2 + 8 = 10$

$(4, 10)$

$y - b = m(x - a)$

$y - 10 = \frac{7}{4}(x - 4)$

Q3 A patient is injected with 156 ml of a drug. Every 8 hours, 22% of the drug passes out of his bloodstream. To compensate, a further 25 ml dose is given every 8 hours.

(a) Find a recurrence relation for the amount of drug in his bloodstream.

(b) Calculate the amount of drug remaining after 24 hours.

(a) $U_{n+1} = 0.78U_n + 25 \quad U_0 = 156$

(b) $U_1 = 0.78(156) + 25 = 146.68$

$U_2 = 0.78(146.68) + 25 = 139.4$

$U_3 = 0.78(139.4) + 25 = 133.7$ mL

Today we will be learning about stationary points and their nature.

Homework Online due 29.9.15

Differentiation HW:

Q8. If $f(x) = \dfrac{x-3}{x^2\sqrt{x}}$ find the gradient of the tangent of f(x) at the point where x = 1

$$f(x) = \frac{x-3}{x^2 x^{\frac{1}{2}}} = \frac{x-3}{x^{\frac{5}{2}}} = x^{-\frac{5}{2}}(x^1-3) = x^{-\frac{3}{2}} - 3x^{-\frac{5}{2}}$$

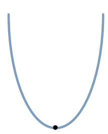
$$f'(x) = -\frac{3}{2}x^{-\frac{5}{2}} + \frac{15}{2}x^{-\frac{7}{2}}$$

$$= \frac{-3}{2\sqrt{x^5}} + \frac{15}{2\sqrt{x^7}}$$

$$f'(1) = \frac{-3}{2\sqrt{1^5}} + \frac{15}{2\sqrt{1^7}} = \frac{-3}{2} + \frac{15}{2} = \frac{12}{2} = \underline{\underline{6}}$$

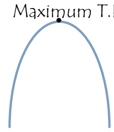
Stationary points 23.9.15

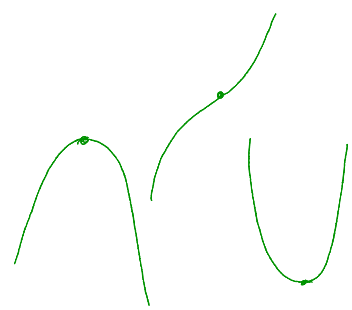
The stationary points (turning points) on a curve are where the gradient of the curve is equal to zero. $m = \frac{dy}{dx} = 0$



Minimum T.P.
S.P.

S.P.
Maximum T.P.

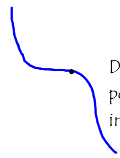




Stationary points

A point of inflection is the point on the curve where the curvature changes (but continues to increase or decrease).

Increasing point of inflection



Decreasing point of inflection

Stationary points & their nature

To find the nature of a stationary point:

- First differentiate the function and equate to zero to find the T.P's
- Then consider f'(x) near each stationary point by using values of x that are slightly bigger or slightly smaller.
- If f'(x) > 0, then the graph is increasing
- If f'(x) < 0, then the graph is decreasing
- Create a nature table to show your results
- If it is a point of inflection, the graph will be increasing or decreasing for both smaller and bigger values.

Daily Practice 24.9.15

Q1. Find the equation of the line perpendicular to $4x - 1 + 2y = 0$ that passes through the point (3, 4)

$y = -2x + \frac{1}{2}$
$m = -2$
$\perp m = \frac{1}{2}$

$y - b = m(x - a)$
$y - 4 = \frac{1}{2}(x - 3)$
$2y - 8 = x - 3$
$x - 2y + 5 = 0$

Q2. State the centre and the radius of the circle

$x^2 + y^2 - 12x + 8y - 12 = 0$

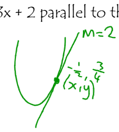
$2g = -12$ $\quad 2f = 8$
$g = -6$ $\quad f = 4$

Centre $= (6, -4)$
Radius $= \sqrt{g^2 + f^2 - c} = \sqrt{64} = \underline{\underline{8}}$

Q3. Find the equation of the tangent to $y = x^2 + 3x + 2$ parallel to the line $y = 2x + 1$

$\underline{\underline{m = 2}}$

$dy/dx = 2x + 3 = m$
$2x + 3 = 2$
$2x = -1$
$x = -\frac{1}{2}$



$y = (-\frac{1}{2})^2 + 3(-\frac{1}{2}) + 2$
$y = \frac{1}{4} - \frac{3}{2} + 2$
$y = \frac{1}{4} - \frac{6}{4} + \frac{8}{4}$
$y = \frac{3}{4}$

$(-\frac{1}{2}, \frac{3}{4})$

$y - b = m(x - a)$
$\underline{\underline{y - \frac{3}{4} = 2(x + \frac{1}{2})}}$

Today we will be continuing to learn about stationary points and their nature.

Homework Online due 29.9.15

Stationary points & their nature

To find the nature of a stationary point:

- First differentiate the function and equate to zero to find the T.P's S.P.'s
- Then consider f'(x) near each stationary point by using values of x that are slightly bigger or slightly smaller.
- If f'(x) > 0, then the graph is increasing
- If f'(x) < 0, then the graph is decreasing
- Create a nature table to show your results
- If it is a point of inflection, the graph will be increasing or decreasing for both smaller and bigger values.

Stationary points & their nature

Example:

Find the Stationary Points on the following curves and determine their nature.

1) $y = 4x^2 - 16x + 7$

At a S.P. $dy/dx = m = 0$

$dy/dx = 8x - 16 = 0$
$8x = 16$
$\underline{\underline{x = 2}}$

$y = 4(2)^2 - 16(2) + 7$
$y = 16 - 32 + 7$
$\underline{\underline{y = -9}}$

S.P. $= (2, -9)$

x	2^- (1)	2	2^+ (3)
dy/dx	$-$	0	$+$
Slope	\	—	/

Nature Table

$\underline{\underline{(2, -9) \text{ minimum S.P.}}}$

Daily Practice 25.9.15

Q1. An organic gardener discovers that 250 aphids are born each week in the greenhouse during the summer. She introduces a parasite that kills 50% of the aphids each week. If there were 700 aphids at the beginning of the summer:

(a) Write down a recurrence relation that describes this situation

$U_{n+1} = 0.5U_n + 250$

(b) The gardener says that in the long run if the number of aphids exceeds 200, then some of the plants will die. Is there a chance that this will happen?

$L = \frac{b}{1-a} = \frac{250}{1-0.5} = \frac{250}{0.5} = \underline{\underline{500}}$

Yes, the plants will die as $500 > 200$

Today we will be continuing to learn about stationary points and increasing/decreasing functions.

Stationary points & their nature

2) $y = x(x - 3)^2 = x(x^2 - 6x + 9) = x^3 - 6x^2 + 9x$

$dy/dx = m = 3x^2 - 12x + 9 = 0$

$x^2 - 4x + 3 = 0$

$(x - 3)(x - 1) = 0$

$x = 3$ and $x = 1$

$y = 3(3-3)^2$, $\underline{y = 0}$ $\quad$ $y = 1(1-3)^2$, $\underline{y = 4}$

S.P.'s = (3, 0) (1, 4)

x	3^- (2)	3	3^+ (4)	1^- (0)	1	1^+ (2)
dy/dx	−	0	+	+	0	−
Slope	\	—	/	/	—	\

(3,0) = minimum S.P.

(1,4) = maximum S.P.

Page 106 Ex. 6M Q1, 3, 5, 7, 9

Questions

① $9 - x^2 = y$ ② $3x - x^3$

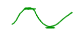
Increasing and Decreasing Functions

Examples

2)

~~3)~~ For $y = {}^2/_3x^3 - 2x$, find the intervals in which the function is increasing and decreasing

$dy/dx = 2x^2 - 2 = 0$ at S.P.'s



$2x^2 - 2 = 0$

$2(x^2 - 1) = 0$

$x^2 - 1 = 0$

$x^2 = 1$

$\underline{x = \pm 1}$

$y = \frac{2}{3}(1)^3 - 2(1)$

$y = -\frac{4}{3}$ $\quad$ $y = \frac{2}{3}(-1)^3 - 2(-1)$

S.P's = $(1, -\frac{4}{3})$, $(-1, \frac{4}{3})$

x	-1^- (−2)	−1	-1^+ (0)	1^- (0)	1	1^+ (2)
dy/dx	+	0	−	−	0	+
Slope	/	—	\	\	—	/

Max. at $(-1, \frac{4}{3})$ Min. at $(1, -\frac{4}{3})$

$\boxed{x > 1 \quad x < -1 \text{ Increasing}}$ $\quad$ $\boxed{-1 < x < 1 \text{ decreasing}}$

Ex. 6M Q4, 7

6L Q2. (b) (e) (h)

$x^4 - 4x^3 = 0$

$x^3(x - 4) = 0$

$x^3 = 0 \quad x = 4$

$x = 0 \quad x = 4$

Increasing and Decreasing Functions

Examples

4) For $y = 3x^2 - 4x$, find the intervals in which the function is increasing and decreasing

Stationary points & their nature

3) $y = 8x^3 - 3x^4$

Daily Practice **28.9.15**

1. Find the derivative of

(a) $y = x^2 + 3\sqrt{x}$

$y = x^2 + 3x^{\frac{1}{2}}$

$dy/dx = 2x + \frac{3}{2}x^{-\frac{1}{2}}$

$= 2x + \frac{3}{2x^{\frac{1}{2}}}$

$= \underline{\underline{2x + \frac{3}{2\sqrt{x}}}}$

(b) $f(x) = \frac{x^2 - 4}{\sqrt{x}}$

$f(x) = \frac{x^2 - 4}{x^{\frac{1}{2}}}$

$= x^{-\frac{1}{2}}(x^2 - 4)$

$= x^{\frac{3}{2}} - 4x^{-\frac{1}{2}}$

$f'(x) = \frac{3}{2}x^{\frac{1}{2}} + 2x^{-\frac{3}{2}}$

$f'(x) = \underline{\underline{\frac{3\sqrt{x}}{2} + \frac{2}{\sqrt{x}^3}}}$

(c) $y = \frac{(x - 2)(x + 1)}{\sqrt{x}}$

$y = \frac{x^2 + x - 2x - 2}{x^{\frac{1}{2}}}$

$y = \frac{x^2 - x - 2}{x^{\frac{1}{2}}}$

$y = x^{-\frac{1}{2}}(x^2 - x - 2)$

$y = x^{\frac{3}{2}} - x^{\frac{1}{2}} - 2x^{-\frac{1}{2}}$

$dy/dx = \frac{3}{2}x^{\frac{1}{2}} - \frac{1}{2}x^{-\frac{1}{2}} + 1x^{-\frac{3}{2}}$

$dy/dx = \underline{\frac{3\sqrt{x}}{2} - \frac{1}{2\sqrt{x}} + \frac{1}{\sqrt{x}^3}}$

Today we will be continuing to practise questions on increasing & decreasing functions and Stationary Points.

Homework Due Tomorrow

Daily Practice 29.9.2015

Q1. Find the limit of the recurrence relation $u_{n+1} = 0.5u_n + 12$

$L = \frac{b}{1-a} = \frac{12}{0.5} = \underline{24}$ $\quad -1 < 0.5 < 1 \Rightarrow$ limit exists

Q2. The point (–5, 3) lies on the circumference of the circle $x^2 + y^2 + ax - 6y + 4 = 0$. Find the value of a

$(-5)^2 + 3^2 + a(-5) - 6(3) + 4 = 0$

$25 + 9 - 5a - 18 + 4 = 0$

$20 = 5a$

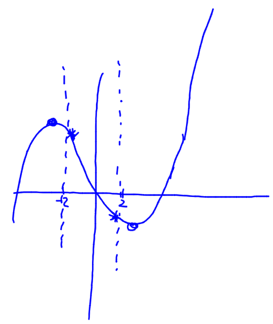
$\underline{4 = a}$

Q3. Find $A'(r)$ when $A(r) = 3\pi r^2 + \frac{800}{r}$

$A(r) = 3\pi r^2 + 800r^{-1}$

$A'(r) = 6\pi r - 800r^{-2}$

$A'(r) = 6\pi r - \frac{800}{r^2}$



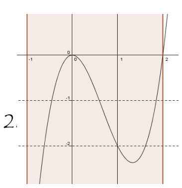
Today we will be learning to read off maximum and minimum values from closed intervals.

Homework due today

Closed Intervals – Maximum and Minimum 29.9.15

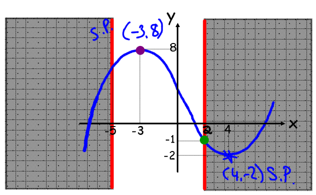
A closed interval or restricted domain means looking at just part of a graph instead of the whole thing.

For example, if given the interval $-1 \leq x \leq 2$, then you are only looking at the graph between the points where x = –1 and x = 2. Draw a vertical line at each point.



Closed Intervals Max. & Min. Values

The maximum or minimum values are the max. and min. y – coordinates that occur on the graph. They are either at the end points of the closed interval or at the stationary points.



On the closed interval $-5 \leq x \leq 2$

- the max value = 8
- the min value = –1

If the closed interval is extended to $-5 \leq x \leq 5$ the min value would change to –2 (S.P)

Closed Intervals Max. & Min. Values

Example:

Find the maximum & minimum values of the function $f(x) = 2x^3 - 3x^2 - 12x + 5$ within the closed interval $-2 \leq x \leq 1$.

$f'(x) = 6x^2 - 6x - 12 = 0$ at S.P's

$x^2 - x - 2 = 0$

$(x + 1)(x - 2) = 0$

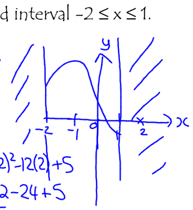
$x = -1 \qquad x = 2$

$y = 2(-1)^3 - 3(-1)^2 - 12(-1) + 5 \qquad y = 2(2)^3 - 3(2)^2 - 12(2) + 5$

$y = -2 - 3 + 12 + 5 \qquad y = 16 - 12 - 24 + 5$

$y = 12 \qquad y = -15$

$\underline{(-1, 12)}$ $\quad$ S.P.s $\quad$ $\underline{(2, -15)}$ ← not in inteval



$f(1) = 2(1)^3 - 3(1)^2 - 12(1) + 5 = \underline{-8}$ minimum value

$f(-2) = 2(-2)^3 - 3(-2)^2 - 12(-2) + 5 = 1$

$\Rightarrow \underline{12}$ = maximum value

Daily Practice 30.9.2015

Q1. Find the equation of the tangent to the curve y = $3x^2 - 3$ at the point where x = –1

$dy/dx = m = 6x$ at $x = -1$

$6(-1) = \underline{-6 = m}$

$y = 3(-1)^2 - 3$

$y = 0$

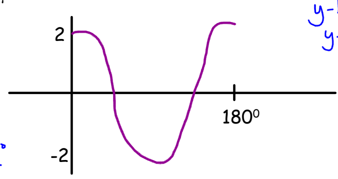
$\underline{(-1, 0)}$

$y - b = m(x - a)$

$y - 0 = -6(x + 1)$

$\underline{\underline{y = -6x - 6}}$

Q2. State the equation of the function shown



$y = a\cos bx$

$\underline{\underline{y = 2\cos 2x^\circ}}$

Today we will be learning how to sketch curves using differentiation.

Homework Online due 6.10.15

Before Differentiation, what did we need to sketch the graph of a function?

Turning Point(s) + Nature

y-intercept $(x = 0)$

Roots $(y = 0)$

Curve Sketching

To sketch the graph of a function, you will need to find out:

- The x and y – intercepts
- The Stationary Points and their nature. (Differentiation)
- The behaviour of the curve for large positive and negative x (as x –> –∞ and as x –> +∞)

The above information (1 & 2) needs to be clearly annotated on the graph.

Curve Sketching

Examples

1) Sketch and annotate fully:

y = $x^2(x - 4)$

Roots $\Rightarrow y = 0$

$x^2(x - 4) = 0$

$x^2 = 0$ $\quad x - 4 = 0$

$(0,0)$ $\quad (4,0)$

y-intercept $\rightarrow x = 0$

$y = 0^2(0 - 4)$

$y = 0$

$(0,0)$

At S.P.'s $dy/dx = 0$

$y = x^2(x - 4) = x^3 - 4x^2$

$dy/dx = 3x^2 - 8x = 0$

$x(3x - 8) = 0$

$x = 0$ $\quad 3x - 8 = 0$

$3x = 8$

$x = \frac{8}{3}$

$y = 0^2(0 - 4)$

$y = 0$

$(0,0)$

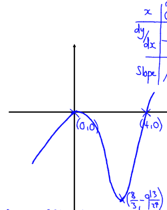
$y = \frac{8}{3}^2\left(\frac{8}{3} - 4\right)$

$y = -9\frac{13}{27}$

$\left(\frac{8}{3}, -9\frac{13}{27}\right)$

Nature

x	0^- (-1)	0	0^+ (1)	$\frac{8}{3}^-$ (2)	$\frac{8}{3}$	$\frac{8}{3}^+$ (3)
dy/dx	+	0	—	—	0	+
Slope	/	—	\	\	—	/



$(0,0)$ = max. S.P.

$\left(\frac{8}{3}, -9\frac{13}{27}\right)$ = minimum S.P.

$y = x^3 - 4x$

as $x \rightarrow +\infty$

$y = x^3 - 4x \rightarrow +\infty$

as $x \rightarrow -\infty$

$y = x^3 - 4x \rightarrow -\infty$

Question: Sketch $x^3 + 3x^2$

Daily Practice 1.10.2015

Q1. State the turning point and its nature of the function y = $4(x + 5)^2 - 2$

T.P = $(-5, -2)$ minimum

Q2. (a) Differentiate with respect to x, the function

$$A(x) = x^2 + \frac{432000}{x} = x^2 + 432000x^{-1}$$

(b) Work out the stationary point(s) of the above function and state the nature

$A'(x) = 2x - 432000x^{-2} = 0$

$2x - \frac{432000}{x^2} = 0$

$2x^3 - 432000 = 0$

$2x^3 = 432000$

$x^3 = 216000$

$x = \sqrt[3]{216000}$

$\underline{\underline{x = 60}}$

$A(60) = (60)^2 + \frac{432000}{60}$

$= 10800$

$(60, 10800)$

x	60^-	60	60^+
dy/dx	—	0	+
slope	\	—	/

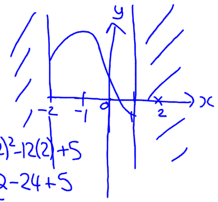
$\underline{\underline{\text{minimum}}}$

Closed Intervals Max. & Min. Values

Example:

Find the maximum & minimum values of the function
$f(x) = 2x^3 - 3x^2 - 12x + 5$ within the closed interval $-2 \le x \le 1$.

$f'(x) = 6x^2 - 6x - 12 = 0$ at S.P's
$x^2 - x - 2 = 0$
$(x + 1)(x - 2) = 0$
$x = -1 \quad x = 2$



$y = 2(-1)^3 - 3(-1)^2 - 12(-1) + 5$
$y = -2 - 3 + 12 + 5$
$y = 12$
$(-1, 12)$

$y = 2(2)^3 - 3(2)^2 - 12(2) + 5$
$y = 16 - 12 - 24 + 5$
$y = -15$
$(2, -15) \leftarrow$ not in interval

S.P.s

$\underline{12}$ = maximum value

$f(1) = 2(1)^3 - 3(1)^2 - 12(1) + 5 = \underline{-8}$ minimum value

2(f)

$f(x) = 3x - x^3$

$f(-2) = 3(-2) - (-2)^3 = -6 + 8 = 2$
$f(1) = 3(1) - (1)^3 = 2$

$f'(x) = 3 - 3x^2 = 0$
$3 = 3x^2$
$x^2 = 1$
$x = \pm 1$

$f(1) = 2 \quad f(-1) = 3(-1) - (-1)^3$
$= -3 + 1 = -2$

SP's $= (1, 2) \quad (-1, -2)$

$\underline{\text{Max.} = 2}$ $\underline{\text{min.} = -2}$

Today we will be learning to sketch the curve of the derived function.

Homework Online due 6.10.15

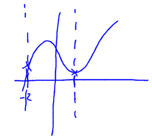
Closed Intervals Max. & Min. Values

- Find endpoints first
- Then find S.P.'s

Question - Closed Intervals.ggb

Question:

Find the maximum and minimum values of $y = 8x^3 - 3x^2$ in the interval $-2 \le x \le 1$



End-points:
$f(-2) = 8(-2)^3 - 3(-2)^2 = -76$
$f(1) = 8(1)^3 - 3(1)^2 = 5$

S.P's $y = 8x^3 - 3x^2$
$dy/dx = 24x^2 - 6x = 0$
$6x(4x - 1) = 0$
$6x = 0 \quad 4x - 1 = 0$
$x = 0 \quad x = \frac{1}{4}$
$(0, 0) \quad (\frac{1}{4}, -\frac{1}{16})$

$f(0) = 8(0)^3 - 3(0)^2 = 0$
$f(\frac{1}{4}) = 8(\frac{1}{4})^3 - 3(\frac{1}{4})^2 = -\frac{1}{16}$

Maximum value $= \underline{5}$ minimum value $= \underline{-76}$

Ex. 60 Q1(b) Q2. (f)

Daily Practice 2.10.2015

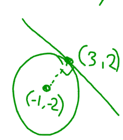
Q1. A circle C_1 has equation $x^2 + y^2 + 2x + 4y - 27 = 0$.

(a) Write down the centre and calculate the radius of C_1.

(b) The point P(3, 2) lies on the circle C_1.
Find the equation of the tangent at P.

(a) $(-1, -2)$ Radius $= \sqrt{1^2 + 2^2 + 27} = \sqrt{32} = \underline{4\sqrt{2}}$

(b)



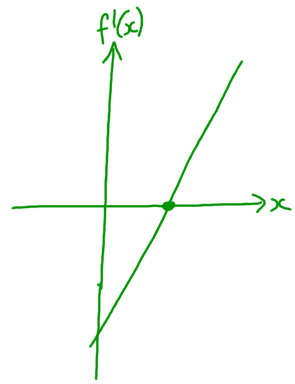
$m = \frac{2 - (-2)}{3 - (-1)} = \frac{4}{4} = 1 \quad \perp m = -1$

$y - 2 = -1(x - 3)$
$\underline{y = -x + 5}$

Sketching the graph of the derived function

The graph of $f'(x)$ shows the value of the gradient at each point on $y = f(x)$.

- Only focus on the S.P's and the shape of the graph.
- All S.P.'s on the graph become roots on the graph of the derivative (because $dy/dx = m = 0$ here)
- Wherever the curve is strictly decreasing, the derivative is negative so the graph of the derivative will lie below the x - axis.
- Whenever the graph is stricly increasing, the derivative is positive so the graph of the derivative will lie above the x - axis.



→ T.P → -m

at T.P ⇒ m=0

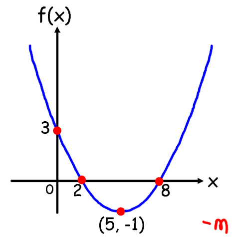
after T.P ⇒ +m

Sketching the graph of the derived function

Examples

Each graph shows the graph of a function, y = f(x). Sketch the graph of y = f'(x) , the derived function.

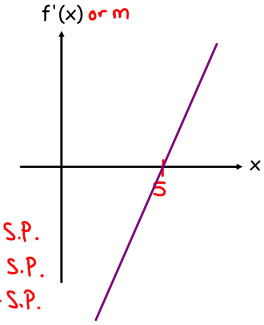
1)



-m until S.P.

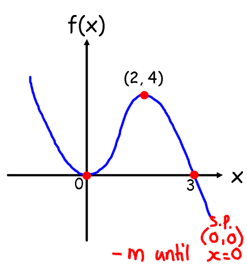
m=0 at S.P.

+m after S.P.



Sketching the graph of the derived function

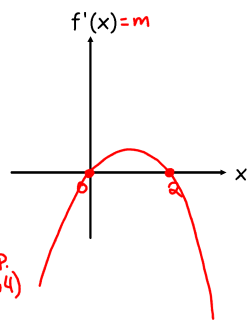
2)



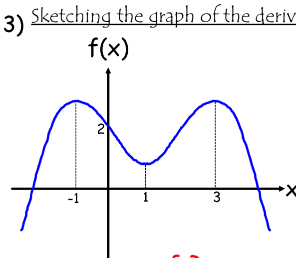
-m until x=0

+m after (0,0) until (2,4) S.P.

-m after (2,4) [x=2]



3) Sketching the graph of the derived function

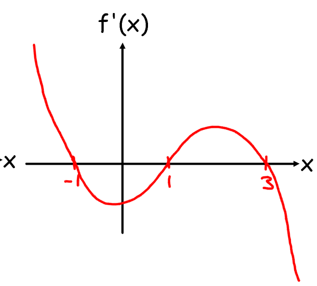


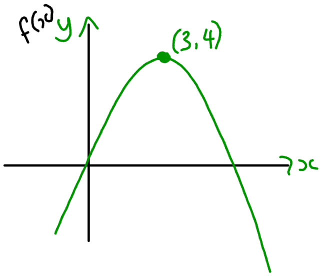
+m until x=-1 S.P.

-m after x=-1 until x=1 S.P.

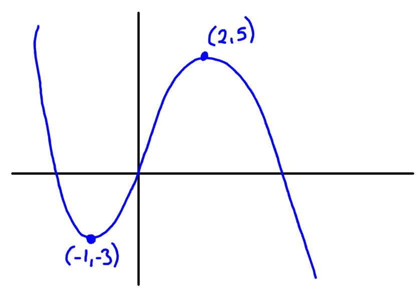
+m after x=1 S.P. until x=3 S.P.

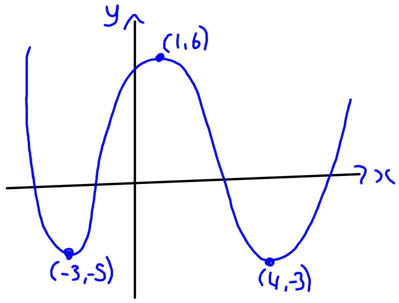
-m after x=3



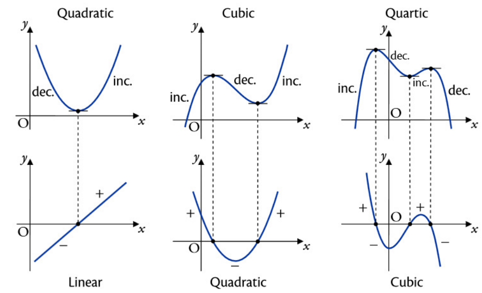


wipeboards





Sketching the graph of the derived function



Daily Practice **5.10.2015**

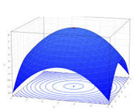
A curve has equation $y = x^3 - 3x^2$.

(a) Find where this curve cuts the x and y axes.
(b) Find the stationary points of the curve and determine their nature.
(c) Sketch the curve.

(

Today we will be learning about optimisation.

Optimisation



Optimisation is a process that uses differentiation to find the minimum and maximum of a function to solve problems in context.

Optimisation

Example: The diagram shows a rectangular cake-box with no top. It has been made from thin card. The volume of the box is 500cm³. The base of the box is a square with sides of length x cm.

(a) Show that the area A cm² of card used used to make this box is given by $A = x^2 + \frac{2000}{x}$

$A = x^2 + 4xh$

$A = x^2 + 4x\left(\frac{500}{x^2}\right)$

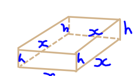
$A = x^2 + \frac{2000}{x}$

$V = 500$

$x \times x \times h = 500$

$x^2 h = 500$

$h = \frac{500}{x^2}$



(b) Find the minimum area of card used.

$A(x) = x^2 + 2000x^{-1}$

$A'(x) = 2x - 2000x^{-2} = 0$

$2x - \frac{2000}{x^2} = 0$

$2x^3 - 2000 = 0$

$2x^3 = 2000$

$x^3 = 1000$

$x = \sqrt[3]{1000}$

$x = 10$

x	10^-	10	10^+
dy/dx	$-$	0	$+$
slope	\	—	/

$\Rightarrow$ Minimum at $x = 10$

Area $= x^2 + \frac{2000}{x}$

$A = 10^2 + \frac{2000}{10}$

$A = 100 + 200$

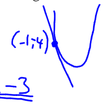
$A = 300\text{cm}^2$

Pg 112 Ex.6Q

Daily Practice 6.10.2015

Q1. State the equation of the tangent to the function $y = 3x^2 + 7x$ at the point where $x = -1$

$y = 3(-1)^2 + 7(-1)$
$y = -4$



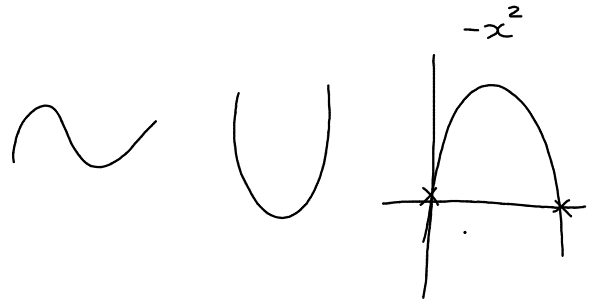
$m = dy/dx = 6x + 7$
$= 6(-1) + 7 = 1$

$y + 4 = 1(x + 1)$
$\underline{y = x - 3}$

Q2. Given the function $f(x) = \sqrt{x}(x + \frac{3}{x^2})$ find $f'(x)$

$f(x) = x^{\frac{1}{2}}(x + 3x^{-2})$
$f(x) = x^{\frac{3}{2}} + 3x^{-\frac{3}{2}}$
$f'(x) = \frac{3}{2}x^{\frac{1}{2}} - \frac{9}{2}x^{-\frac{5}{2}}$
$\underline{f'(x) = \frac{3}{2}\sqrt{x} - \frac{9}{2\sqrt{x}^5}}$

$3x^{-\frac{3}{2}}$
$\frac{3}{1} \times \frac{-3}{2}$



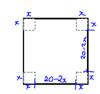
Today we will be continuing to practise questions on optimisation.

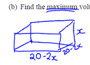
Homework due today.

Examples:

A square piece of card of side 20 cm has a square of side x cm cut from each corner.
An open box is formed by turning up the sides.

(a) Show that the volume of the box can be written as
$V = 400x - 80x^2 + 4x^3$
(b) Find the maximum volume of the box.





$V = L \times B \times H$
$V = (20-2x)(20-2x)x$
$V = (400 - 80x + 4x^2)x$
$V = \underline{400x - 80x^2 + 4x^3}$

(b) $0 \le x \le 10$

End points: $V(0) = \underline{0}$
$V(10) = 400(10) - 80(10^2) + 4(10)^3 = \underline{0}$

$V'(x) = 400 - 160x + 12x^2 = 0$
$12x^2 - 160x + 400 = 0$
$3x^2 - 40x + 100 = 0$
$(3x - 10)(x - 10) = 0$
$3x - 10 = 0$ $\quad x - 10 = 0$
$x = \frac{10}{3}$ $\quad x = 10$

x	(3) $\frac{10}{3}^-$	$\frac{10}{3}$	(4) $\frac{10}{3}^+$	(9) 10^-	10	(11) 10^+
dy/dx	+	0	−	−	0	+
Slope	/	—	\	\	—	/

Max. at $\underline{x = \frac{10}{3}}$ $\qquad$ Min. at $\underline{x = 10}$

$V = 400x - 80x^2 + 4x^3$
$V(\frac{10}{3}) = 400(\frac{10}{3}) - 80(\frac{10}{3})^2 + 4(\frac{10}{3})^3 = \underline{\frac{16000}{27}\text{ cm}^3}$ max. volume.

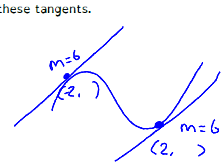
Ex. 6Q

Daily Practice 7.10.2015

Going through the homework.

Q5. A curve has equation $y = x^3 - 6x$. There are two tangents to the curve that have a gradient of 6. Find the equations of each of these tangents.

$m = 6$
$m = dy/dx = 3x^2 - 6 = 6$
$+6 \quad +6$
$3x^2 = 12$
$x^2 = 4$
$x = \pm 2$

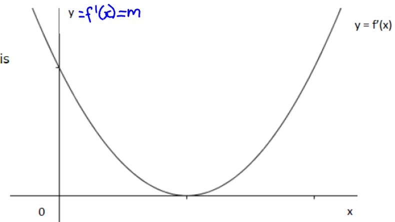


$y = 2^3 - 6(2)$
$y = -4$
$(2, -4)$

$y = (-2)^3 - 6(-2)$
$y = -8 + 12$
$y = 4$ $\quad (-2, 4)$

$\underline{y + 4 = 6(x - 2)}$ $\qquad$ $\underline{y - 4 = 6(x + 2)}$

Q6. The diagram shows the graph of y = f'(x). The x – axis is a tangent to this graph. Explain why the function f(x) is never decreasing



$m \geq 0$ because $f'(x)$ always ≥ 0

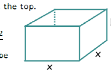
$\Rightarrow$ Graph is always increasing

Q7. A storage tank in the shape of a cuboid has a capacity of 108m³. It has a square base of side x metres with vertical sides and is open at the top.

(i) Express the height, h in terms of x.

(ii) Show that the surface area, S is given by $S = x^2 + \frac{432}{x}$

(iii) Find the dimensions of the tank if the surface area is to be a minimum.



(i) $V = 108m^3$

$V = L \times B \times H$

$V = x \times x \times h = x^2h = 108$

$h = \frac{108}{x^2}$

(ii) $x^2 + xh + xh + xh + xh$

$x^2 + 4xh$

$x^2 + 4x\left(\frac{108}{x^2}\right)$

$x^2 + \frac{432x}{x^2}$

$S = x^2 + \frac{432}{x}$

(iii) $S = x^2 + \frac{432}{x} = x^2 + 432x^{-1}$

$S'(x) = 2x - 432x^{-2} = 0$

$= 2x - \frac{432}{x^2} = 0$

$2x^3 - 432 = 0$

$2x^3 = 432$

$x^3 = 216$

$x = \sqrt[3]{216} = 6$

x	6^-	6	6^+
dy/dx	$-$	0	$+$
slope	\	—	/

minimum

Dimensions = $6 \times 6 \times 3$

$h = \frac{108}{6^2} = 3$

Ex. 6Q, 6R

Q3

6R - Q2+5

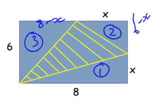
Pg. 117

Q21, 22

Daily Practice 8.10.15

(a) Show that the area of the triangle is given by the equation

$$H(x) = 24 - 4x + \frac{1}{2}x^2$$



Area $\square = 6 \times 8 = 48$ units²

$A_1 = \frac{1}{2}(8x) = 4x$

$A_2 = \frac{1}{2}x(6-x) = 3x - \frac{x^2}{2}$

$A_3 = \frac{1}{2}6(8-x) = 24 - 3x$

$4x + 3x - \frac{x^2}{2} + 24 - 3x$

$= 4x - \frac{x^2}{2} + 24$

$48 - \left(4x - \frac{x^2}{2} + 24\right) = 48 - 4x + \frac{x^2}{2} - 24$

$= 24 - 4x + \frac{x^2}{2}$

(b) Hence find the greatest and least possible values of the area of the triangle

$A(x) = 24 - 4x + \frac{1}{2}x^2$

$A'(x) = -4 + x = 0$

$x = 4$

x	4^- (3)	4	4^+ (5)
dy/dx	$-$	0	$+$
slope	\	—	/

minimum T.P at $x = 4$

$A(4) = 24 - 4(4) + \frac{1}{2}(4)^2 = 16$ units²

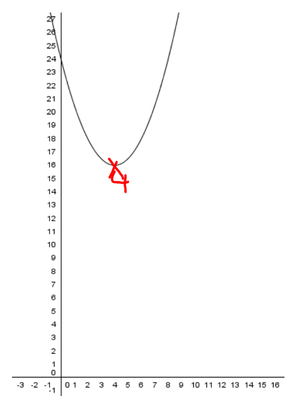
Endpoints : $0 \leq x \leq 6$

$A(0) = 24 - 4(0) + \frac{1}{2}(0)^2 = 24$ units²

$A(6) = 24 - 4(6) + \frac{1}{2}(6^2) = 18$ units²

max possible area = 24 units² at $x = 0$

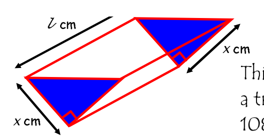
least possible area = 16 units² at $x = 4$



New Higher Marking Scheme!!

**Note: Ensure that the straight line equation is multiplied out and simplified to gain full marks.

Homework Online due 27.10.15



This open water tank is in the shape of a triangular prism. It has a capacity of 108 litres.

a) Show that the surface area of this prism is given by

$$A(x) = x^2 + \frac{432000}{x} \qquad (3)$$

b) Find the value of x which minimises this surface area. (5)