

Differentiation (2) Homework Solutions

Total = 30 marks

Q1.  $y = (3x-2)(2x+4) = 6x^2 + 12x - 4x - 8 = 6x^2 + 8x - 8$  ✓  
 $\frac{dy}{dx} = 12x + 8$  ✓

3/3

$x=2 \Rightarrow 12(2) + 8 = 24 + 8 = 32$  ✓

Q2.  $f(x) = (2x - \frac{4}{x})^2 \quad f'(-2)$

$f(x) = 4x^2 - 16 + \frac{16}{x^2} = 4x^2 - 16 + 16x^{-2}$  ✓

$f'(x) = 8x - 32x^{-3}$  ✓

$= 8x - \frac{32}{x^3}$

$f'(-2) = 8(-2) - \frac{32}{(-2)^3} = -16 + 4 = -12$  ✓

4/4

Q3.  $y = x^3 + 2x^2 + 3x - 1$  at  $(-1, -3)$

$\frac{dy}{dx} = m$  ✓

$= 3x^2 + 4x + 3$  ✓ at  $x = -1$

$\Rightarrow 3(-1)^2 + 4(-1) + 3$

$= 3 - 4 + 3$

$= \underline{2}$  ✓

$y - b = m(x - a)$

$y + 3 = 2(x + 1)$  ✓

$y + 3 = 2x + 2$

$2x - y - 1 = 0$

4/4

Q4.  $f(x) = 2x^2 + 5x + 4$  at  $x = -1$

$f'(x) = 4x + 5$  ✓

$m = f'(x)$

at  $x = -1 \Rightarrow 4(-1) + 5 = 1 = m$  ✓

$y - b = m(x - a)$

$y - 1 = 1(x + 1)$  ✓

$y - 1 = x + 1$

$x - y + 2 = 0$

$y = 2(-1)^2 + 5(-1) + 4$

$= 2 - 5 + 4$

$= 1$  ✓ pt. =  $(-1, 1)$

4/4

Q5.  $y = x^3 - 6x$  2 tangents that have  $m = 6$

$$\frac{dy}{dx} = m$$

$$\Rightarrow \frac{dy}{dx} = 3x^2 - 6 = 6$$

$$3x^2 = 12$$

$$x^2 = 4$$

$$x = \pm 2$$

$$x = 2$$

$$y = (2)^3 - 6(2)$$

$$y = 8 - 12 = -4$$

$$(2, -4)$$

$$x = -2$$

$$y = (-2)^3 - 6(-2)$$

$$y = -8 + 12$$

$$y = 4 \quad (-2, 4)$$

6/6

$$y + 4 = 6(x - 2)$$

$$y + 4 = 6x - 12$$

$$6x - y - 16 = 0$$

$$y - 4 = 6(x + 2)$$

$$y - 4 = 6x + 12$$

$$6x - y + 16 = 0$$

Q6. The function is never decreasing because  $\frac{dy}{dx}$  or  $f'(x)$  is never less than zero, because the graph isn't negative for  $f'(x)$ .  
Or gradient never less than zero.

Q7. Capacity =  $108\text{m}^3$

(i)  $V = L \times B \times H$

$$108 = x \times x \times h$$

$$108 = x^2 h$$

$$h = \frac{108}{x^2}$$

(ii) S.A =  $x^2 + xh + xh + xh + xh$

$$= x^2 + 4xh$$

$$= x^2 + 4x \left( \frac{108}{x^2} \right) = x^2 + \frac{432}{x}$$

$$S(x) = x^2 + 432x^{-1}$$

(iii)  $S'(x) = 2x - 432x^{-2} = 0$

$$2x - \frac{432}{x^2} = 0$$

$$2x^3 - 432 = 0$$

$$2x^3 - 432 = 0$$

$$2x^3 = 432$$

$$x^3 = 216$$

$$x = \sqrt[3]{216} = 6$$

$$h = \frac{108}{6^2}$$

$$\frac{108}{6^2} = 3$$

$x$	$6^-$	$6$	$6^+$
$\frac{dy}{dx}$	$-$	$0$	$+$
slope	$\searrow$	$ $	$\swarrow$

minimum

8/8