

Functions HW Solutions

Higher

Total = 27 marks

Q1.  $f(x) = 2x - 5 \quad \{-3, \leq x \leq 3\}$

$f(-3) = -11 \quad f(-2) = -9 \quad f(-1) = -7 \quad f(0) = -5 \quad f(1) = -3 \quad f(2) = -1 \quad f(3) = 1$

$\{-11, -9, -7, -5, -3, -1, 1\}$  ✓

Q2.  $f(x) = \frac{x-2}{5}$

$f(k) = 2$

$\frac{k-2}{5} = 2$  ✓

$k-2 = 10$

$k = 12$  ✓

Q3.  $f(x) = x^2 + 2 \quad g(x) = 2x + 3$

$f(g(x)) = (2x+3)^2 + 2$  ✓  
 $= 4x^2 + 12x + 9 + 2$   
 $= 4x^2 + 12x + 11$  ✓

$g(f(x)) = 2(x^2+2) + 3$  ✓  
 $= 2x^2 + 4 + 3$   
 $= 2x^2 + 7$  ✓

Q4.  $f(x) = 6x^2 - 4x$

$g(x) = \frac{1}{3x-6}$

(a)  $g(f(x)) = \frac{1}{3(6x^2-4x)-6} = \frac{1}{18x^2-12x-6} = \frac{1}{6(3x^2-2x-1)} = \frac{1}{6(3x+1)(x-1)}$  ✓

(b)  $g(f(x))$  domain  $\{x \in \mathbb{R}, x \neq -1/3, 1\}$  ✓

Q5.  $f(x) = (x-1)(x+3) \quad g(x) = x^2 + 3$

$f(g(x)) = (x^2+3-1)(x^2+3+3)$  ✓  
 $= (x^2+2)(x^2+6)$   
 $= x^4 + 8x^2 + 12$  ✓

$g(g(x)) = (x^2+3)^2 + 3$  ✓  
 $= x^4 + 6x^2 + 9 + 3$   
 $= x^4 + 6x^2 + 12$  ✓

$x^4 + 8x^2 + 12 - (x^4 + 6x^2 + 12)$   
 $= x^4 + 8x^2 + 12 - x^4 - 6x^2 - 12$  ✓  
 $= 2x^2$  ✓

Q6.  $f(x) = \frac{4}{x+2}$

$g(x) = \frac{2}{x} - 2$

$f(g(x)) = \frac{4}{(\frac{2}{x}-2)+2} = \frac{4}{2/x} = 4 \div \frac{2}{x} = 4 \times \frac{x}{2} = 4x/2 = 2x$  ✓

Q7.  $f(x) = 3x - 10 \quad g(x) = 4 - 2x \quad h(x) = 1/6(2-x)$

(a)  $k(x) = f(g(x)) = 3(4-2x) - 10 = 12 - 6x - 10 = 2 - 6x$  ✓

(b)  $h(k(x)) = 1/6(2 - (2 - 6x)) = 1/6(6x) = x$  ✓

(c)  $h$  is the inverse of  $k$  ✓

Q8. (i)  $f^{-1}(x) = \sqrt[3]{x-2}$

