Q1. Differentiate

(a) $y = (5x - 2)^3$

$$\frac{dy}{dx} = \underbrace{3(5x-2)^2}(5)$$

$$= 15(5x-2)^2$$

2

(b) $y = \frac{2}{5x+2} = 2(5x+2)^{-1}$

$$\frac{dy}{dx} = \underbrace{-2(5x+2)^{-2}}(5) = -10(5x+2)^{-2}$$

$$= \frac{-10}{(5x+2)^2}$$

3

(c) $y = 3\sin 2x$

~~7 marks~~

$dy/dx = 6\cos 2x$

2

Q2. Integrate

(a) $\int\left(\sqrt{6x+1}\right)dx = \int (6x+1)^{\frac{1}{2}}\,dx$

$$= \left[\frac{(6x+1)^{3/2}}{3/2(6)} + C\right]$$

$$= \frac{(6x+1)^{3/2}}{9} + C$$

4

$$= \frac{1}{9}(6x+1)^{3/2} + C$$

$$= \frac{1}{9}\left(\sqrt{6x+1}\right)^{3} + C$$

(b) $\int_1^2 \frac{8}{(1-2x)^3}dx$ $= \int_1^2 8(1-2x)^{-3}\,dx$

$$= \left[\frac{8(1-2x)^{-2}}{(-2)(-2)} \right]_1^2 = \left[\frac{8}{4(1-2x)^2} \right]_1^2$$

$$= \left[\frac{8}{4(1-4)^2} \right] - \left[\frac{8}{4(1-2)^2} \right]$$

$$= \frac{2}{9} - 2 = \underline{\underline{\frac{-16}{9}}}$$

4

(c) $\int 4cosxdx$

~~10 marks~~

$$= 4\sin x + c$$

2

Q3. Find the rate of change of the function $f(x) = 4\sin^3 x$ when $x = \frac{5\pi}{6}$

3 marks

$f'(x) = 12\sin^2 x(\cos x)$

$= 12\left(\sin\left(\frac{5\pi}{6}\right)\right)^2 \cos\left(\frac{5\pi}{6}\right)$

$= 12\left(\frac{1}{2}\right)^2\left(-\frac{\sqrt{3}}{2}\right)$

$= \frac{-12\sqrt{3}}{8} = \frac{-3\sqrt{3}}{2}$

$4(\sin x)^3$

$12(\sin x)^2 \cos x$

$\sin\frac{5\pi}{6} = \sin 150° = \sin 30° = \frac{1}{2}$

$\cos\frac{5\pi}{6} = -\cos\frac{\pi}{6} = -\frac{\sqrt{3}}{2}$

3

Q4. A curve has the equation $y = x^2 - 4x + 7$, find the equation of the tangent to the curve when $x = 5$

3 marks

$dy/dx = 2x - 4$

$x = 5 \Rightarrow 2(5) - 4 = 6 = m$

$y = 5^2 - 4(5) + 7$

$y = 12 \quad (5, 12)$

$y - 12 = 6(x - 5)$

3

Q5. The gradient of a tangent to a curve is given by $\frac{dy}{dx} = 3\cos 2x$.

The curve passes through the point $\left(\frac{7\pi}{6}, \sqrt{3}\right)$. 4 marks

Find y in terms of x.

$$\int 3\cos 2x \, dx$$

$$y = \tfrac{1}{2}(3)\sin 2x + C$$

$$y = \tfrac{3}{2}\sin 2x + C$$

$$\sqrt{3} = \tfrac{3}{2}\sin 2\left(\tfrac{7\pi}{6}\right) + C$$

$$\sqrt{3} = \tfrac{3}{2}\sin \tfrac{7\pi}{3} + C$$

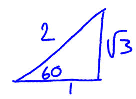
$$\sqrt{3} = \tfrac{3}{2}\left(\tfrac{\sqrt{3}}{2}\right) + C$$

$$\sqrt{3} = \frac{3\sqrt{3}}{4} + C$$

$$\sqrt{3} - \frac{3\sqrt{3}}{4} = C$$

$$\frac{4\sqrt{3} - 3\sqrt{3}}{4} = C$$

$$\frac{\sqrt{3}}{4} = C$$



$$\frac{7\pi}{3} = 420^\circ$$
$$-360^\circ$$
$$60^\circ$$

$$\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$$

$$y = \tfrac{3}{2}\sin 2x + \frac{\sqrt{3}}{4}$$

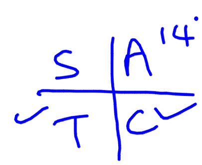
4

Q6. Solve 2sin2x + cosx = 0 where 0 ≤ x ≤ 360 4 marks

$$2(2\sin x\cos x) + \cos x = 0$$

$$4\sin x\cos x + \cos x = 0$$

$$\cos x(4\sin x + 1) = 0$$



$$\cos x = 0 \qquad 4\sin x = -1$$

$$x = 90^\circ, 270^\circ \qquad \sin x = -1/4$$

$$x = 194^\circ, 346^\circ$$

(*a*) The expression $3\sin x - 5\cos x$ can be written in the form $R\sin(x+a)$ where $R > 0$ and $0 \le a < 2\pi$.

Calculate the values of R and a. **4**

(*b*) Hence find the value of t, where $0 \le t \le 2$, for which

$$\int_0^t (3\cos x + 5\sin x)\,dx = 3.$$ **7**

(a) $R\sin(x+a) = R\sin x\cos a + R\cos x\sin a$

$R\cos a = 3 \qquad R\sin a = -5$

$\tan a = \frac{-5}{3}$

S | A
T | C

$a = \tan^{-1}\left(\frac{5}{3}\right) = 1.03$ radians

$a = 2\pi - 1.03 = 5.25$ radians

4

$R = \sqrt{3^2 + (-5)^2} = \sqrt{34}$

$\sqrt{34}\sin(x + 5.25)$

(b) $\int_0^t (3\cos x + 5\sin x)\,dx = 3 \qquad 0 \le t \le 2$

$\left[3\sin x - 5\cos x\right]_0^t = 3$

$\left[3\sin t - 5\cos t\right] - \left[3\sin 0 - 5\cos 0\right] = 3$

$3\sin t - 5\cos t - (-5) = 3$

$3\sin t - 5\cos t = -2$

S | A 0.35
T | C

$\sqrt{34}\sin(t + 5.25) = -2$

$\sin(t + 5.25) = -0.3429971\!7$

$\sin^{-1}(0.343) = 0.35$

$t + 5.25 = 3.49$ or $t + 5.25 = 5.93$

$t = \cancel{-1.76}$ or $t = 0.68$

$\pi + 0.35 = 3.49 \qquad 2\pi - 0.35 \quad 5.93$

$0 \le t \le 2$

7

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