Integration HW Solutions 32 marks Higher

Q1. (a) $\int \frac{1}{6x^2}\,dx = \int \frac{1}{6}x^{-2}\,dx = \frac{\frac{1}{6}x^{-1}}{-1} + c = \frac{-1}{6x} + c$

3/3

3/3 (b) $\int x^3(x+2)\,dx = \int x^4 + 2x^3\,dx = \frac{x^5}{5} + \frac{2x^4}{4} + c = \frac{x^5}{5} + \frac{x^4}{2} + c$

(c) $\int \frac{2x^3+3}{4}\,dx = \int \frac{1}{2}x^3 + \frac{3}{4}\,dx = \frac{\frac{1}{2}x^4}{4} + \frac{3}{4}x + c$

3/3

$= \frac{x^4}{8} + \frac{3x}{4} + c$

Q2. $\int_0^3 (3x+2)\,dx = \left[\frac{3x^2}{2} + 2x\right]_0^3 = \left[\frac{3(3)^2}{2} + 2(3)\right] - [0]$

3/3

$= \frac{27}{2} + 6 = \frac{39}{2}$

(b) $\int_{-1}^{5} (3x+4)^2\,dx = \int_{-1}^{5} 9x^2 + 24x + 16\,dx = \left[\frac{9x^3}{3} + \frac{24x^2}{2} + 16x\right]_{-1}^{5}$

4/4

$\Rightarrow \left[3x^3 + 12x^2 + 16x\right]_{-1}^{5} \Rightarrow \left[3(5)^3 + 12(5)^2 + 16(5)\right] - \left[3(-1)^3 + 12(-1)^2 + 16(-1)\right]$

$\Rightarrow 755 - [-7] = 762$

(c) $\int_1^{16} \frac{\sqrt{x} - 4}{\sqrt{x}}\,dx = \int_1^{16} \frac{x^{\frac{1}{2}} - 4}{x^{1/2}}\,dx = \int_1^{16} x^{-1/2}(x^{1/2} - 4)\,dx$

$= \int_1^{16} x^0 - 4x^{-1/2}\,dx = \left[x - \frac{4x^{1/2}}{1/2}\right]_1^{16} = \left[x - 8x^{1/2}\right]_1^{16}$

4/4

$= \left[16 - 8\sqrt{16}\right] - \left[1 - 8\sqrt{1}\right] = [16 - 32] - [1 - 8] = -16 + 7 = -9$

Q3. $y = 4x - x^2$

Roots $\Rightarrow y = 0$ ✓

$-x^2 + 4x = 0$

$x^2 - 4x = 0$

5/5

$x(x-4) = 0$

$x = 0 \quad x = 4$ ✓

$$\int_0^4 4x - x^2 \, dx$$

$$\Rightarrow \left[\frac{4x^2}{2} - \frac{x^3}{3}\right]_0^4 = \left[2x^2 - x^3/3\right]_0^4$$

$$= \left[2(4)^2 - 4^3/3\right] - [0]$$

$$= 32 - 64/3 = 32/3 \text{ units}^2$$ ✓

Q4. (a) $y = 8 - 2x - x^2$

$x^2 + 2x - 8 = 0$ ✓

$(x - 2)(x + 4) = 0$

$x = 2 \quad x = -4$

$A(-4,0) \quad B(2,0)$ ✓

7/7

(b) $$\int_{-4}^{2} 8 - 2x - x^2 \, dx + \int_2^3 8 - 2x - x^2 \, dx$$ ✓ ✓

$$\Rightarrow \left[8x - \frac{2x^2}{2} - \frac{x^3}{3}\right]_{-4}^{2} + \left[8x - \frac{2x^2}{2} - \frac{x^3}{3}\right]_2^3$$ ✓

$$\Rightarrow \Big[\left[8(2) - (2)^2 - 2^3/3\right] - \left[8(-4) - (-4)^2 - \frac{(-4)^3}{3}\right]\Big] + \Big[\left[8(3) - (3)^2 - 3^3/3\right] - \left[8(2) - (2)^2 - 2^3/3\right]\Big]$$

$$\Rightarrow \Big[\left[28/3\right] - \left[-80/3\right]\Big] + \Big[[6] - \left[28/3\right]\Big]$$ ✓

$$\Rightarrow 108/3 + 10/3 = 118/3 \text{ units}^2$$ ✓

Q5 + 6 Optional

Q5. $f'(x) = 4x(x-1)$ $f(-1) = 2$, formula for $f(x)$ — 10 marks

$$\int 4x^2 - 4x \, dx$$

$$\Rightarrow \frac{4x^3}{3} - \frac{4x^2}{2} + c = f(x)$$ ✓ $$\Rightarrow f(x) = \frac{4x^3}{3} - 2x^2 + c$$

$$f(-1) = \frac{4(-1)^3}{3} - 2(-1)^2 + c$$ ✓ ✓

$$2 = -\frac{4}{3} - 2 + c$$

$$c = \frac{16}{3}$$ ✓

$$\Rightarrow f(x) = \frac{4x^3}{3} - 2x^2 + \frac{16}{3}$$ ✓

Q6.

curve $y = 5x - x^2$

line $x + y - 8 = 0 \Rightarrow y = -x + 8$

between $x = 2$ and $x = 4$

Upper Curve − Lower

$$\Rightarrow \int_2^4 [5x - x^2] - [-x + 8] \, dx$$

$$= \int_2^4 5x - x^2 + x - 8 \, dx$$

$$= \int_2^4 -x^2 + 6x - 8 \, dx = \left[\frac{-x^3}{3} + \frac{6x^2}{2} - 8x\right]_2^4$$

$$\Rightarrow \left[\frac{-4^3}{3} + \frac{6(4)^2}{2} - 8(4)\right] - \left[\frac{-2^3}{3} + \frac{6(2)^2}{2} - 8(2)\right]$$

$$\Rightarrow \left[-64/3 + 48 - 32\right] - \left[-8/3 + 12 - 16\right]$$

$$= \left[-16/3\right] - \left[-20/3\right] = -16/3 + 20/3 = 4/3 \text{ units}^2$$