

Higher Hw Solutions - The Circle

Q1) Eq. of circle centre (3,1) $r=2$

$$(x-3)^2 + (y-1)^2 = 4$$

Q2) Centre (0,0), passes through (-4,1)

$$\begin{aligned} x^2 + y^2 &= r^2 \\ (-4)^2 + (1)^2 &= r^2 \\ 16 + 1 &= r^2 \\ 17 &= r^2 \end{aligned}$$

$$x^2 + y^2 = 17$$

Q3) (3,5) (-1,1) Find equation diameter

$$\begin{aligned} \text{Mid-point} = \text{centre} &= \left(\frac{3-1}{2}, \frac{5+1}{2} \right) \\ &= (1, 3) \end{aligned}$$

Radius = $\frac{1}{2}$ diameter = $\frac{1}{2}$ distance between endpoints

$$\begin{aligned} d &= \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2} \\ &= \sqrt{(-1-3)^2 + (1-5)^2} = \sqrt{16+16} = \sqrt{32} \\ &= 4\sqrt{2} \end{aligned}$$

$$4\sqrt{2} \div 2 = 2\sqrt{2} = r \quad r^2 = 8$$

$$\text{Equation } \circ (x-1)^2 + (y-3)^2 = 8$$

$$\text{Q4. } x^2 + y^2 - 4x + 8y - 5 = 0$$

$$\text{Centre} = (2, -4)$$

$$\text{Radius} = \sqrt{(2)^2 + (-4)^2 + 5} = \sqrt{25} = 5$$

Q5. a(k,2) b(-b,-k) endpoints. centre = c(3,-5). Find k

centre = midpoint ab

$$\Rightarrow \left(\frac{k-b}{2}, \frac{2-k}{2} \right) = (3, -5)$$

$$x_2 \frac{k-b}{2} = 3 \times 2$$

$$k-b = 6 \quad \underline{k = 12}$$

(ii) Radius = $\frac{1}{2}$ distance ab a(12,2) b(-6,-12)

$$\begin{aligned} d &= \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2} = \sqrt{(-6-12)^2 + (-12-2)^2} \\ &= \sqrt{(-18)^2 + (-14)^2} = \sqrt{520} = \sqrt{4 \times 130} = 2\sqrt{130} \\ r &= 2\sqrt{130} \div 2 = \underline{\sqrt{130}} \end{aligned}$$

Q6. $x^2 + y^2 - 4x - 8y + 4 = 0$
centre = (2,4) radius = $\sqrt{2^2 + 4^2 - 4} = \sqrt{16} = 4$

$$\text{Equation of line } l \circ 4x + 2 = 8 \Rightarrow \underline{y = 8}$$

Q7. ① $y = -1$

$$\text{② } x^2 + y^2 - 6x + 10y + 2 = 0$$

Sub ① into ②

$$\begin{aligned} x^2 + (-1)^2 - 6x + 10(-1) + 2 &= 0 \\ x^2 + 1 - 6x - 10 + 2 &= 0 \\ x^2 - 6x - 7 &= 0 \\ (x + 1)(x - 7) &= 0 \\ x = -1 & \quad x = 7 \\ y = -1 & \quad y = -1 \\ \underline{(-1, -1)} & \quad \underline{(7, -1)} \end{aligned}$$

Q8. $x^2 + y^2 - 6x - 2y - 3 = 0$ Eq. tangent at (5,4)
centre = (3,1)

$$\text{① } (5,4) \quad m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-1}{5-3} = \frac{3}{2} \quad \perp m = -\frac{2}{3}$$

$$\begin{aligned} \text{Equation } \circ y - 4 &= -\frac{2}{3}(x - 5) \\ 3y - 12 &= -2x + 10 \\ 2x + 3y - 22 &= 0 \end{aligned}$$

Q9. P is midpoint of AB

$$\begin{aligned} A(-3, -2) \\ B(3, 6) \quad \text{Mid} &= \left(\frac{-3+3}{2}, \frac{-2+6}{2} \right) = (0, 2) \end{aligned}$$

$$\begin{aligned} \text{(b) length } ab &= d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2} \\ &= \sqrt{(3+3)^2 + (6+2)^2} = \sqrt{36+64} = \sqrt{100} = \underline{10} \end{aligned}$$

