

Higher Maths - Expressions and Formulae Revision Questions

Outcome 1.1 Applying algebraic skills to logarithms and exponentials

1. Simplify fully

(a) $\log_4 2 + \log_4 8$

(b) $\log_3 108 - \log_3 4$

(c) $\log_3 18 - \log_3 2$

(d) $\log_5 100 - \log_5 4$

(e) $\log_4 8 + \log_4 8$

(f) $2\log_{10} 2 + 2\log_{10} 5$

(g) $\log_9 3 - \log_9 6 + \log_9 18$

(h) $\log_3 9 - \log_3 \frac{1}{3}$

(i) $\frac{1}{2} \log_2 16 - \frac{1}{3} \log_2 8$

2. Solve the logarithmic equations for $x > 0$

(a) $\log_4(x + 3) = 2$

(b) $\log_3(x - 2) = 4$

(c) $\log_6(x - 8) = 2$

(d) $\log_a 4 + \log_a x = \log_a 12$

(e) $2\log_a 3 + \log_a x = \log_a 36$

(f) $\log_a(2x + 1) + \log_a 3x = \log_a 63$

(g) $\log_2 x + \log_2(x - 3) = 2$

(h) $\log_2(x - 1) + \log_2(x + 1) = 3$

(i) $\log_3 6x - \log_3(x - 2) = 2$

3. Given $2\log_m n = \log_m 16 + 1$, show that $n = 4\sqrt{m}$

4. The mass, M grams, of a radioactive isotope after a time of t years, is given by the formula

$$M = M_0 e^{-kt} \text{ where } M_0 \text{ is the initial mass of the isotope.}$$

In 5 years a mass of 10 grams of the isotope is reduced to 8 grams.

(a) Calculate k .

(b) Calculate the half-life of the substance i.e. the time taken for half the substance to decay.

5. A cell culture grows at a rate given by the formula $y(t) = Ae^{kt}$ where A is the initial number of cells and $y(t)$ is the number of cells after t hours.

(a) It takes 24 hours for 500 cells to increase in number to 800. Find k .

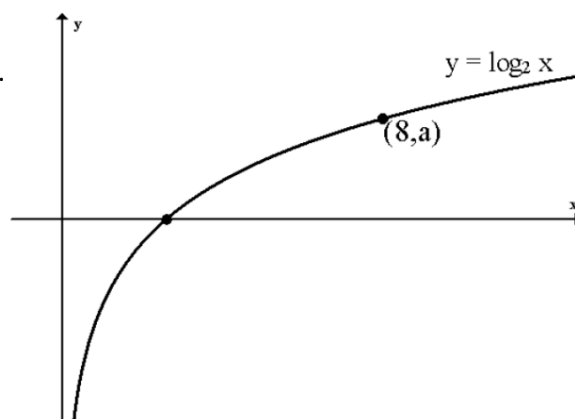
(b) Calculate the time taken for the number of cells to double.

6. The diagram opposite shows the graph of $y = \log_2 x$.

(a) Find the value of a .

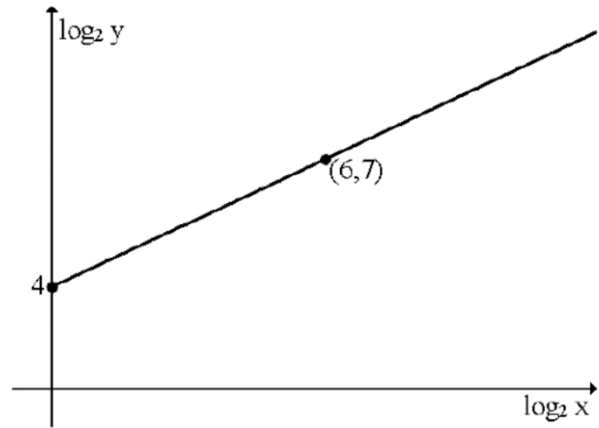
(b) Sketch the graph of $y = \log_2 x - 3$

(c) Sketch the graph of $y = \log_2 4x$



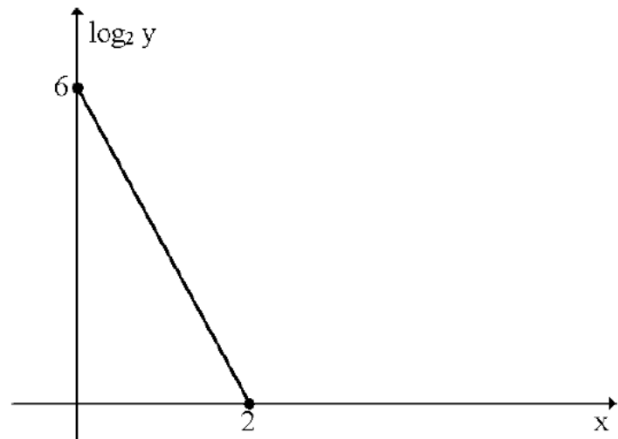
7. The graph opposite illustrates the law $y = kx^n$.

Find the values of k and n .



8. The graph opposite illustrates the law $y = ab^x$.

Find the values of a and b .



9. The concentration of the pesticide, *Xpesto*, in soil can be modelled by the equation $P_t = P_0 e^{-kt}$, where:

- P_0 is the initial concentration;
- P_t is the concentration at time t ;
- t is the time, in days, after the application of the pesticide.

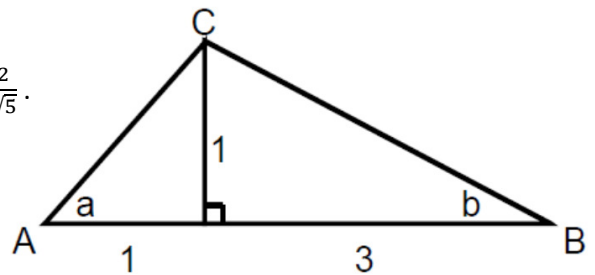
(a) Once in the soil, the half-life of a pesticide is the time taken for its concentration to be reduced to one half of its initial value.

If the half-life of *Xpesto* is 25 days, find the value of k to 2 significant figures.

(b) Eighty days after the initial application, what is the percentage decrease in concentration of *Xpesto*?

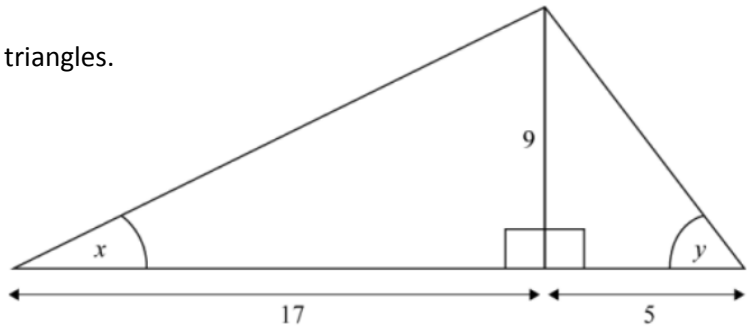
Outcome 1.2 Applying trigonometric skills to manipulating expressions

1. In triangle ABC, show that the exact value of $\sin(a + b)$ is $\frac{2}{\sqrt{5}}$.



2. The diagram shows two right-angled triangles.

Find the exact value of $\sin(x - y)$.



3. Show that $(3 + 2 \cos x)(3 - 2 \cos x) = 4 \sin^2 x + 5$.

4. Show that $(3 + 2 \cos x)(3 - 2 \cos x) = 5 + 4 \sin^2 x$.

5. Express $2 \sin x + 3 \cos x$ in the form $k \sin(x + a)^\circ$ where $k > 0$ and $0 \leq x \leq 360$.

Calculate the values of k and a

6. Express $\cos x - \sin x$ in the form $k \cos(x - \alpha)$, where $k > 0$ and $0 \leq \alpha \leq 360$.

7. (a) The expression can be written in the form $k \sin(x - a)^\circ$, where $k > 0$ and $0 \leq a < 360$.

Calculate the values of k and a .

(b) Determine the maximum value of , where $0 \leq x < 360$.

8. Solve $4\sin x + 3\cos x = 2.5$, $0 \leq x \leq 180$.

9. (a) Diagram 1 shows a right angled triangle, where the line OA has equation $3x - 2y = 0$.

(i) Show that $\tan a = \frac{3}{2}$.

(ii) Find the value of $\sin a$.

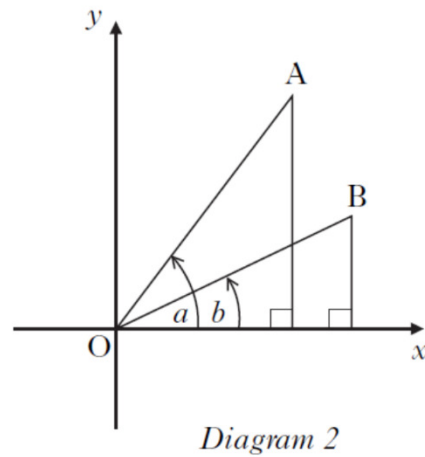
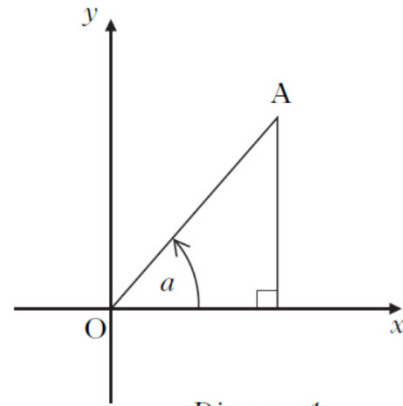
(b) A second right angled triangle is added as shown in Diagram 2.

The line OB has equation $3x - 4y = 0$.

Find the values of $\sin b$ and $\cos b$.

(c) (i) Find the value of $\sin(a - b)$.

(ii) State the value of $\sin(b - a)$.



Outcome 1.3 Applying algebraic and trigonometric skills to functions

1. Sketch the graph of $y = 3\cos(x + \frac{\pi}{4})$ for $0 \leq x \leq 2\pi$.

Show clearly the intercepts on the x-axis and the coordinates of the turning points.

2. The diagram shows the graph of $y = f(x)$ with a maximum turning point at $(-2, 3)$ and a minimum turning point at $(1, -2)$.

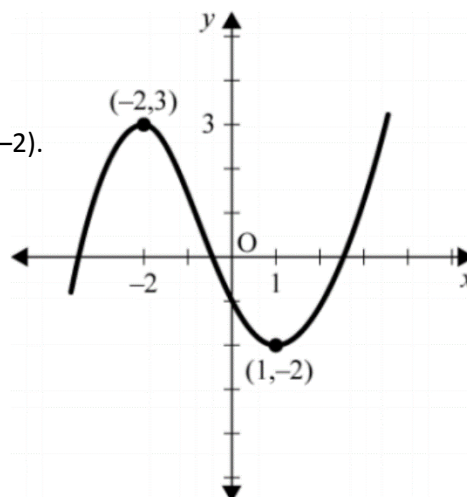
Sketch the graph of

(a) $y = f(x - 2) - 3$.

(b) $y = f(x + 4) + 1$.

(c) $y = 2f(x) + 3$

(d) $y = f(2x) - 2$



3. Sketch the following graphs

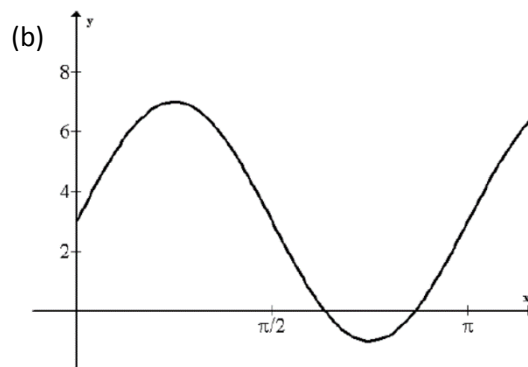
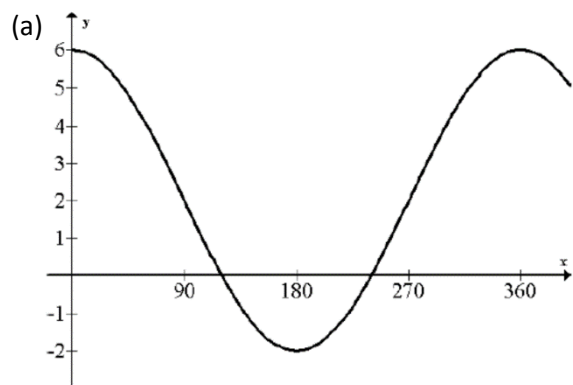
(a) $y = 4\sin x - 1$ $0 \leq x \leq 360$

(b) $y = 4\cos 3x + 1$ $0 \leq x \leq 180$

(c) $y = 2\sin(x - 40)$ $0 \leq x \leq 2\pi$

(d) $y = 3\cos(2x + 30) - 1$ $0 \leq x \leq \pi$

4. Write down the equation of each graph below in the form $y = a\sin bx + c$ or $y = a\cos bx + c$.

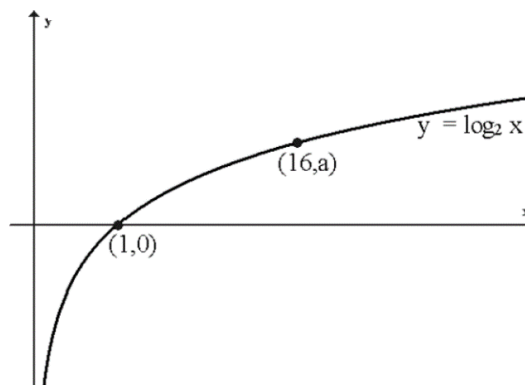


5. The diagram shows part of the graph of $y = \log_2 x$.

(a) Find the value of a .

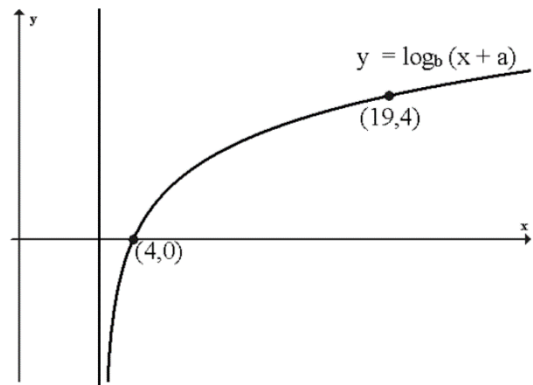
(b) Sketch the graph of $y = \log_2 x - 4$.

(c) Sketch the graph of $y = \log_2 8x$.



6. The diagram shows the graph of $y = \log_b(x + a)$.

Find the values of a and b .



7. $f(x) = 2x^2$ and $g(x) = 5x - 4$.

(a) Find $f(g(2))$.

(b) Find a formula for $f(g(x))$.

8. $f(x) = (x - 1)(x + 3)$ and $g(x) = x^2 + 3$.

Show that $f(g(x)) - g(g(x)) = 2x^2$.

9. The functions f and g , defined on suitable domains, are given by

$$f(x) = \frac{1}{x^2 - 4} \text{ and } g(x) = x + 1$$

(a) Find an expression for $h(x)$, where $h(x) = f(g(x))$.

Give your answer as a single fraction.

(b) State a suitable domain for h .

10. $f(x) = 3x - 2$ and $g(x) = 3x + 2$

(a) Find formulae for $f(g(x))$ and $g(f(x))$.

(b) Find the least value of the product $f(g(x)) \times g(f(x))$.

11. Write down the inverse function

(a) $f(x) = 4x - 5$

(b) $f(x) = \frac{x}{6}$

(c) $f(x) = \frac{2x}{5} + 4$

(d) $f(x) = \frac{2x - 5}{4}$

(e) $f(x) = \frac{4x + 7}{2}$

(f) $f(x) = 12 - \frac{3}{4}x$

(g) $f(x) = \frac{8 - 3x}{13}$

(h) $f(x) = \frac{-3x + 4}{-9}$

Outcome 1.4 Applying geometric skills to vectors

1. An engineer laying flags needs to check that:

- they are in a straight line;
- the distance between Flag 2 and Flag 3 is 3 times the distance between Flag 1 and Flag 2.

Relative to suitable axes, the top-left corner of each flag can be represented by the points A (1, 2, 0), B (4, 0, 2), and C (13, -6, 8) respectively. All three flags point vertically upwards.

A(1,2,0)

B(4,0,2)

C(13,-6,8)



Flag 1



Flag 2

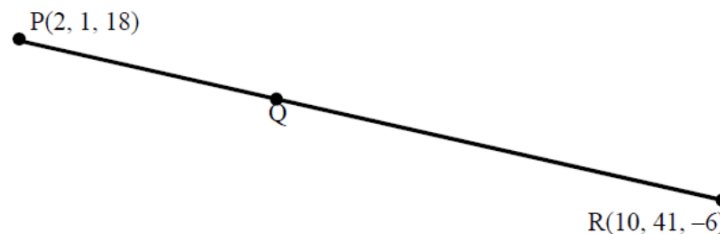


Flag 3

Do the three flags meet the conditions given?

2. The points P, Q and R lie in a straight line, as shown. Q divides PR in the ratio 3:5.

Find the coordinates of the point Q.



3. A is (0,-3,5), B is (7,-6,9) and C is (21,-12,17). Show that A, B and C are collinear stating the ratio AB:BC.

4. $\mathbf{u} = 2\mathbf{i} - 2\mathbf{j} + 4\mathbf{k}$ and $\mathbf{v} = \mathbf{i} + a\mathbf{j} + 7\mathbf{k}$. If $|\mathbf{u}| = |\mathbf{v}|$ find the value of a.

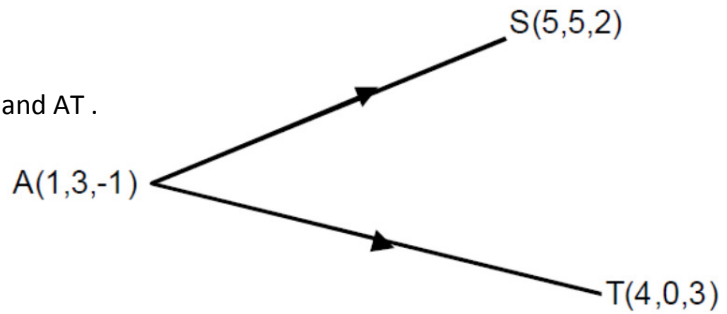
5. A triangle has vertices A(6,-1,9), B(3,-2,11) and C(7,-8,14). Show that this triangle is right-angled at B.

6. A triangle is formed from $R(0,4,-1)$, $S(1,5,2)$ and $T(6,1,-2)$.

(a) Find the vectors \overrightarrow{RS} and \overrightarrow{RT} .

(b) Evaluate $\overrightarrow{RS} \cdot \overrightarrow{RT}$

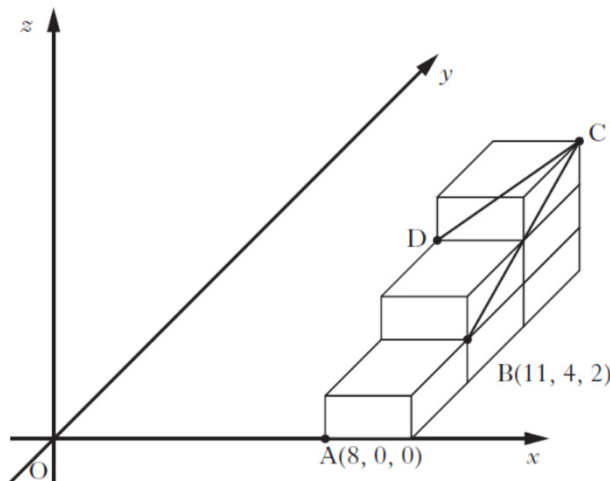
(c) What can you deduce about the lines RS and RT ?



7. (a) For the diagram opposite find \overrightarrow{AS} and \overrightarrow{AT} .

(b) Hence calculate angle TAS .

8. Six identical cuboids are placed with their edges parallel to the coordinate axes as shown in the diagram.



A and B are the points $(8, 0, 0)$ and $(11, 4, 2)$ respectively.

(a) State the coordinates of C and D .

(b) Determine the components of \overrightarrow{CB} and \overrightarrow{CD} .

(c) Find the size of the angle BCD .

9. (a) (i) Show that the points $A(-7, -8, 1)$, $T(3, 2, 5)$ and $B(18, 17, 11)$ are collinear.

(ii) Find the ratio in which T divides AB .

(b) The point C lies on the x -axis.

If \overrightarrow{TB} and \overrightarrow{TC} are perpendicular, find the coordinates of C .

10. A is the point $(3, -3, 0)$, B is $(2, -3, 1)$ and C is $(4, k, 0)$.

(a) (i) Express \vec{BA} and \vec{BC} in component form.

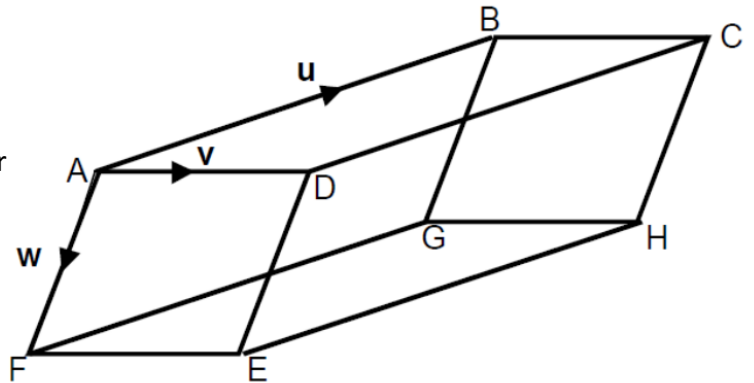
(ii) Show that $\cos \hat{ABC} = \frac{3}{\sqrt{2(k^2+6k+14)}}$

(b) If angle $ABC = 30^\circ$, find the possible values of k .

11. ABCDEFGH is a parallelepiped.

In terms of u , v and w find expressions for

(a) \vec{DC} (b) \vec{HC} (c) \vec{AC} (d) \vec{FD} (e) \vec{CF}



12. The diagram shows the circles with equations

$$(x + 2)^2 + (y + 4)^2 = 100$$

and

$$x^2 + y^2 - 20x - 10y + 100 = 0$$

Find the coordinates of the point P.

