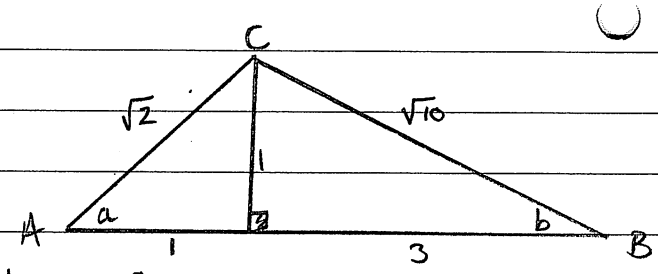
Q11. Show that $\sin(a+b) = \frac{2}{\sqrt{5}}$



$\sin(a+b) = \sin a\cos b + \cos a \sin b$

$= (1/\sqrt{2})(3/\sqrt{10}) + (1/\sqrt{2})(1/\sqrt{10})$

$= \frac{3}{\sqrt{20}} + \frac{1}{\sqrt{20}} = \frac{4}{\sqrt{20}} = \frac{4}{2\sqrt{5}} = \frac{2}{\sqrt{5}}$

Q12. $y = a\cos bx + c$

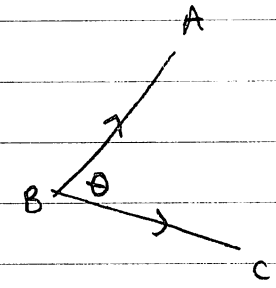
$a = 2$

$b = 2$

$c = -1$

Q9. $A(-1,2,4)$ $B(0,4,2)$ $C(-4,0,2)$

Find $A\hat{B}C$



$$\cos A\hat{B}C = \frac{\vec{BA}\cdot\vec{BC}}{|\vec{BA}||\vec{BC}|}$$

$$\vec{BA} = a - b = \begin{pmatrix}-1\\2\\4\end{pmatrix} - \begin{pmatrix}0\\4\\2\end{pmatrix} = \begin{pmatrix}-1\\-2\\2\end{pmatrix} \qquad \vec{BC} = \begin{pmatrix}-4\\0\\2\end{pmatrix} - \begin{pmatrix}0\\4\\2\end{pmatrix} = \begin{pmatrix}-4\\-4\\0\end{pmatrix}$$

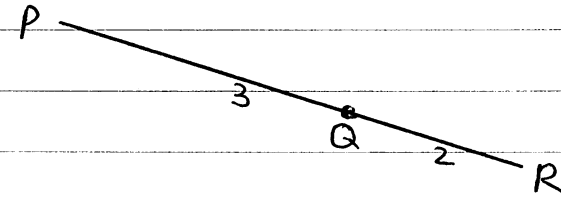
$$\vec{BA}\cdot\vec{BC} = -1(-4) + (-2)(-4) + 2(0) = 4 + 8 = 12$$

$$|\vec{BA}| = \sqrt{(-1)^2 + (-2)^2 + 2^2} = \sqrt{1+4+4} = \sqrt{9} = 3 \qquad |\vec{BC}| = \sqrt{(-4)^2 + (-4)^2 + 0^2} =$$

$$\cos A\hat{B}C = \frac{12}{3\sqrt{32}} = \frac{12}{12\sqrt{2}} = \frac{1}{\sqrt{2}} \qquad A\hat{B}C = \underline{45^\circ}$$

Q10. (a) $P(-2,-1,4)$ $R(7,4,-1)$ Q divides PR in ratio 3:2, find Q

$$PR = r - p = \begin{pmatrix}7\\4\\-1\end{pmatrix} - \begin{pmatrix}2\\-1\\4\end{pmatrix} = \begin{pmatrix}5\\5\\-5\end{pmatrix}$$



$$PQ = \tfrac{3}{5}PR = \tfrac{3}{5}\begin{pmatrix}5\\5\\-5\end{pmatrix} = \begin{pmatrix}3\\3\\-3\end{pmatrix}$$

$$\underline{Q} - \underline{P} = \begin{pmatrix}3\\3\\-3\end{pmatrix} \Rightarrow \begin{pmatrix}x\\y\\z\end{pmatrix} - \begin{pmatrix}2\\-1\\4\end{pmatrix} = \begin{pmatrix}3\\3\\-3\end{pmatrix}$$

$$Q = \begin{pmatrix}5\\2\\1\end{pmatrix} = \underline{(5,2,1) = Q}$$

(b) $T(-1,0,-3)$ $U(-10,-3,-9)$ show Q, T and U are collinear

$$\Rightarrow \vec{QT} = t - q = \begin{pmatrix}-1\\0\\-3\end{pmatrix} - \begin{pmatrix}5\\2\\1\end{pmatrix} = \begin{pmatrix}-6\\-2\\-4\end{pmatrix} \qquad \vec{TU} = u - t = \begin{pmatrix}-10\\-3\\-9\end{pmatrix} - \begin{pmatrix}-1\\0\\-3\end{pmatrix} = \begin{pmatrix}-9\\-3\\-6\end{pmatrix}$$

$\vec{QT} = \tfrac{2}{3}(\vec{TU})$ share a common point and $\vec{QT} = \tfrac{2}{3}\vec{TU} \Rightarrow$ Collinear

QT : TU

3 : 2

Q7. $y = \log_5 x$

$(a, 0)$ $(b, 3)$

$0 = \log_5 a$ $\quad 3 = \log_5 b$

$5^0 = a$ $\quad 5^3 = b$

$\underline{a = 1}$ $\quad \underline{b = 125}$

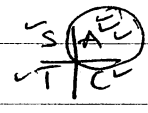
(b) $\cos x + \sqrt{3}\sin x$ $\quad k\cos(x - a)$ $\quad k > 0$ $\quad 0 \leq a \leq 360$

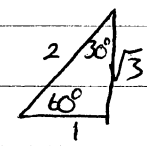
$= k\cos x \cos a + k\sin x \sin a$

$k\cos a = 1$ $\quad k\sin a = \sqrt{3}$

$\tan a = \sqrt{3}/1$

$a = 60°$

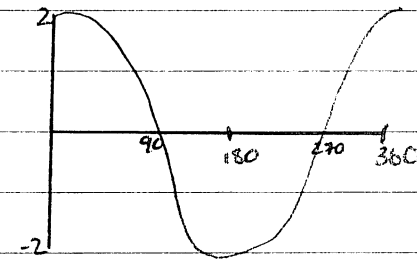


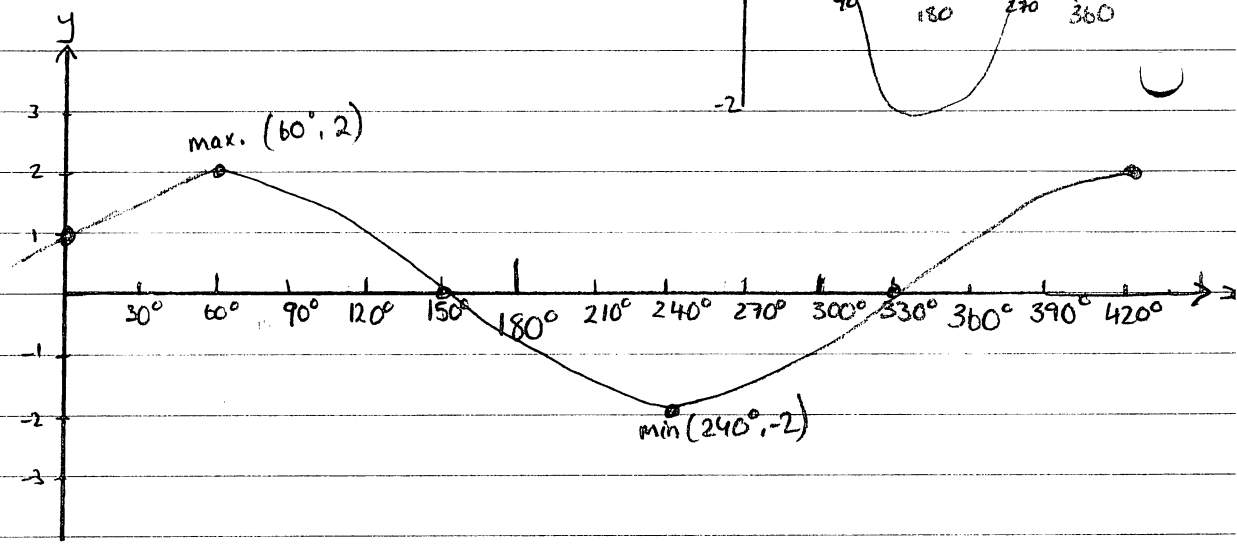


$k = \sqrt{a^2 + b^2} = \sqrt{1^2 + (\sqrt{3})^2} = \sqrt{4} = 2$

$2\cos(x - 60)°$

(b) Sketch $2\cos(x - 60)°$



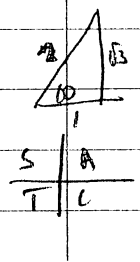


y-intercept $\quad 2\cos(-60) = y$

$2\cos 300 = y$

$2(1/2) = y$

$\underline{y = 1}$



(b) $5\cos^2 x - 2\cos x - 3 = 0$ $\quad 0 \leq x \leq 360°$

$(5\cos x + 3)(\cos x - 1) = 0$

$5\cos x = -3$ $\quad \cos x = 1$

$\cos x = -3/5$ $\quad x = 180°$

$x = 180° - 53° = 147°$

$53°$ (ref.) $\quad x = 180° + 53° = 233°$

S	A
T	C

(c) $3\sin 2x = 3\cos x$ $\quad 0 \leq x \leq 360$

$3\sin 2x - 3\cos x = 0$

$3(2\sin x \cos x) - 3\cos x = 0$

$6\sin x \cos x - 3\cos x = 0$

$3\cos x(2\sin x - 1) = 0$

$3\cos x = 0$ $\quad 2\sin x - 1 = 0$

$x = 180°$ $\quad \sin x = 1/2$

$x = 30°, 150°$

Q5. (a) $\log_9 12 + \log_9 6 - \log_9 8$

$= \log_9 72 - \log_9 8$

$= \log_9 9$

$= 1$

(b) $\frac{2}{3}\log_{10} 8 - \frac{1}{4}\log_{10} 16 + \log_{10} 50$

$= \log_{10}(\sqrt[3]{8})^2 - \log_{10}(\sqrt[4]{16}) + \log_{10} 50$

$= \log_{10} 4 - \log_{10} 2 + \log_{10} 50$

$= \log_{10} 2 + \log_{10} 50$

$= \log_{10} 100 = 2$

Q6. $2\log_x 4 + \log_x 2 = 5$

$\log_x 16 + \log_x 2 = 5$

$\log_x 32 = 5$

$x^5 = 32$

$\underline{x = 2}$

(b) $\frac{3}{4}\log_x 81 - 2\frac{1}{3}\log_x 8 = 3$

$\log_x(\sqrt[4]{81})^3 - \log_x 8^2 = 3$

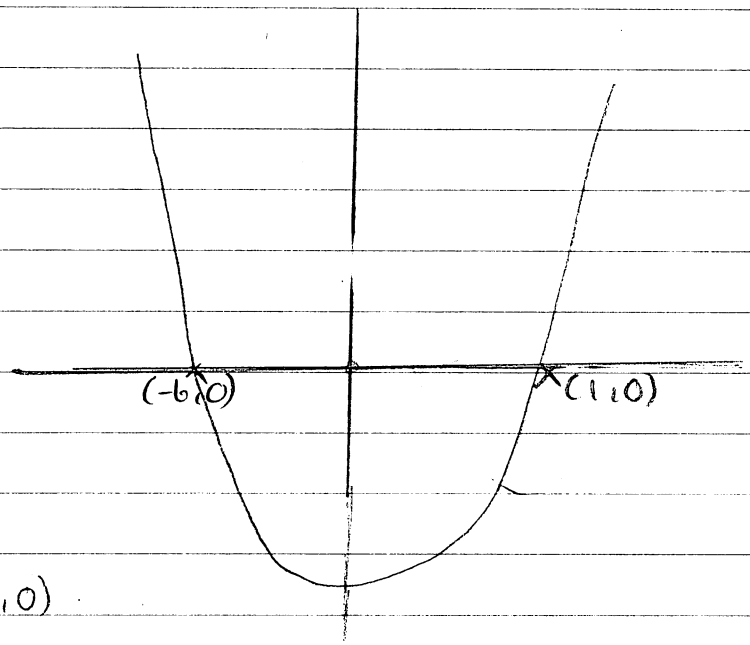
$\log_x 27 - \log_x 64 = 3$

$\log_x 27/64 = 3$

$x^3 = \frac{27}{64}$

$x = \sqrt[3]{\frac{27}{64}} = \underline{\frac{3}{4}}$

(iii) $f'(x)$



$(1,-7) \rightarrow (1,0)$

Q3. $f(x) = 2x-6$ $g(x) = 4-3x$ $h(x) = \frac{1}{6}(2-x)$

(a) $k(x) = f(g(x))$ find $k(x)$

$= 2(4-3x)-6$

$= 8-6x-6 = k(x)$

(b) $h(k(x)) = \frac{1}{6}(2-(8-6x-6))$

$= \frac{1}{6}(2-8+6x+6)$

$= \frac{1}{6}(6x) = x$

(c) k is the inverse of h

Q4. (a) $2\sin 2x - 1 = 0$ $0 \leq x < 360°$

$2\sin 2x = 1$ $0 \leq 2x < 720°$

$\sin 2x = \frac{1}{2}$

S	A
T	C

$\sin^{-1}(\frac{1}{2}) = 30°$

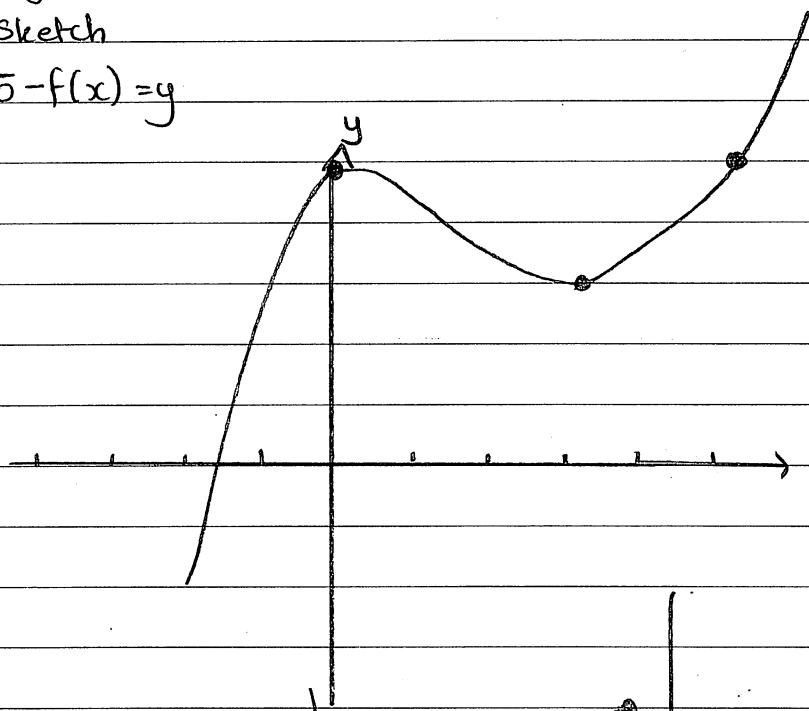
$2x = 30°, 150°, 390°, 510°,$

$x = 15°, 75°, 195°, 255°$

Higher HW solutions — Unit 2 (Expressions and functions) Revision

Sketch

① $5 - f(x) = y$



$-f(x)$: $(3,2) \to (3,-2)$

$(0,0) \to (0,0)$

$(5,0) \to (5,0)$

$5 - f(x)$: $(3,-2) \to (3,3)$

$(0,0) \to (0,5)$

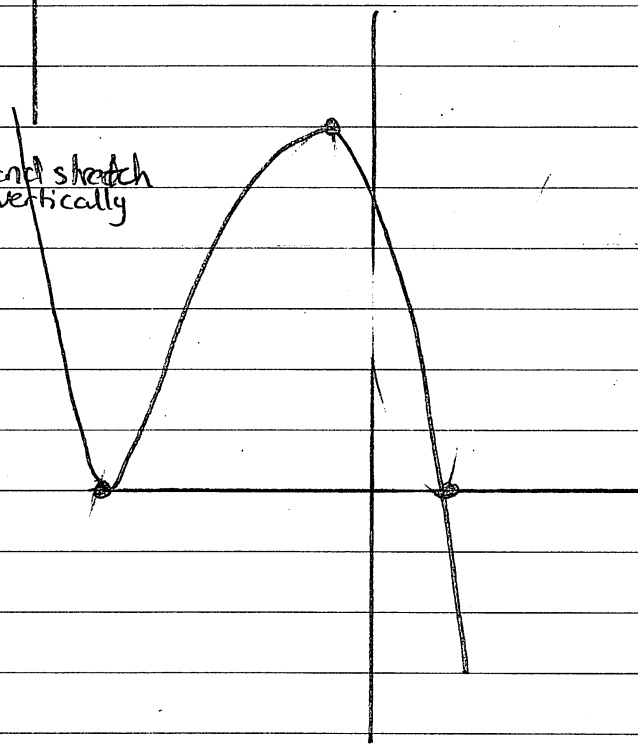
$(5,0) \to (5,5)$

② (i) $-3g(x)$ flip and stretch vertically

$(-6,0) \to (-6,0)$

$(-1,-7) \to (-1,21)$

$(3,0) \to (3,0)$



(ii) $g(x-6)$

$(-6,0) \to (0,0)$

$(1,-7) \to (6,-7)$

$(3,0) \to (9,0)$