Identifying Inverse trig. functions when integrating

Recall the derivatives of $\sin^{-1}x$ and $\tan^{-1}x$

$\sin^{-1}x$	$\dfrac{1}{\sqrt{1-x^2}}$

$\tan^{-1}x$	$\dfrac{1}{1+x^2}$

Identifying Inverse trig. functions when integrating

$\dfrac{1}{\sqrt{a^2-x^2}}$	$\sin^{-1}\left(\dfrac{x}{a}\right)+c$
$\dfrac{1}{a^2+x^2}$	$\dfrac{1}{a}\tan^{-1}\left(\dfrac{x}{a}\right)+c$

Examples: Find the integrals of the following

1. $$\int \frac{1}{\sqrt{9-x^2}}dx = \int \frac{1}{\sqrt{(3)^2-x^2}} = \underline{\sin^{-1}\left(\frac{x}{3}\right)+C}$$

Identifying Inverse trig. functions when integrating

2.
$$\int \frac{1}{16+x^2}\,dx$$

$$= \int \frac{1}{4^2 + x^2}\,dx$$

$$= \underline{\underline{\frac{1}{4}\tan^{-1}\left(\frac{x}{4}\right) + C}}$$

Identifying Inverse trig. functions when integrating

3.
$$\int \frac{1}{\sqrt{5-x^2}}\,dx \quad = \int \frac{1}{\sqrt{(\sqrt{5})^2 - x^2}}\,dx = \underline{\underline{\sin^{-1}\left(\frac{x}{\sqrt{5}}\right) + C}}$$

Identifying Inverse trig. functions when integrating

4. $\int \frac{4}{x^2+36}\,dx = 4\int \frac{1}{6^2+x^2}\,dx = 4\left(\frac{1}{6}\right)\tan^{-1}\left(\frac{x}{6}\right)$

$= \frac{2}{3}\tan^{-1}\left(\frac{x}{6}\right) + C$

Identifying Inverse trig. functions when integrating

5. $\int \frac{1}{\sqrt{25-9x^2}}\,dx = \int \frac{1}{\sqrt{5^2-(3x)^2}}\,dx$

$\downarrow$

$\int \frac{1}{\sqrt{9\left(\frac{25}{9}-x^2\right)}}\,dx$

$= \frac{1}{3}\int \frac{1}{\sqrt{\left(\frac{5}{3}\right)^2-x^2}}\,dx$

$= \frac{1}{3}\sin^{-1}\left(\frac{x}{\frac{5}{3}}\right)+C = \frac{1}{3}\sin^{-1}\left(\frac{3x}{5}\right)+C$

Identifying Inverse trig. functions when integrating

6. $$\int \frac{1}{9+4x^2}dx = \int \frac{1}{4\left(\frac{9}{4}+x^2\right)}dx = \frac{1}{4}\int \frac{1}{\left(\frac{3}{2}\right)^2+x^2}dx$$

$$\frac{1}{4}\left(\frac{1}{3/2}\right)\tan^{-1}\left(\frac{x}{3/2}\right)+C$$

$$= \frac{1}{6}\tan^{-1}\left(\frac{2x}{3}\right)+C$$

Identifying Inverse trig. functions when integrating

7. $$\int_{\frac{3}{2}}^{3} \frac{1}{\sqrt{9-x^2}}dx = \int_{\frac{3}{2}}^{3} \frac{1}{\sqrt{3^2-x^2}}dx = \left[\sin^{-1}\left(\frac{x}{3}\right)\right]_{\frac{3}{2}}^{3}$$

$$\left[\sin^{-1}(1)\right] - \left[\sin^{-1}\left(\frac{3/2}{3}\right)\right] = \sin^{-1}1 - \sin^{-1}\frac{1}{2}$$

$$= \frac{\pi}{2} - \frac{\pi}{6} = \frac{2\pi}{6} = \frac{\pi}{3}$$

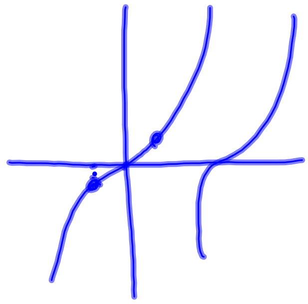
Identifying Inverse trig. functions when integrating

8. $$\int_{-2}^{2}\frac{1}{4+x^2}dx = \int_{-2}^{2}\frac{1}{2^2+x^2}dx$$

$$= \left[\frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)\right]_{-2}^{2} = \left[\frac{1}{2}\tan^{-1}(1)\right] - \left[\frac{1}{2}\tan^{-1}(-1)\right]$$

$$= \frac{1}{2}\left(\frac{\pi}{4}\right) - \frac{1}{2}\left(\frac{-\pi}{4}\right)$$

$$= \frac{\pi}{8} + \frac{\pi}{8} = \underline{\underline{\frac{\pi}{4}}}$$



Identifying Inverse trig. functions when integrating

9. Use the substitution u = x - 3 to find the integral

$$\int\frac{1}{x^2-6x+13}dx$$

$u = x-3$

$\frac{du}{dx} = 1$

$\boxed{du = 1dx}$

$u = x-3$

$\boxed{x = u+3}$

$$\int\frac{1}{(u+3)^2-6(u+3)+13}du$$

$$= \int\frac{1}{u^2+\cancel{6u}+9-\cancel{6u}-18+13}du = \int\frac{1}{u^2+4}du$$

$$= \int\frac{1}{2^2+u^2}du = \frac{1}{2}\tan^{-1}\left(\frac{u}{2}\right)+C$$

$$= \underline{\underline{\frac{1}{2}\tan^{-1}\left(\frac{x-3}{2}\right)+C}}$$

Identifying Inverse trig. functions when integrating

10. Use the substitution $u = 2\sin x$ to evaluate

$$\int_0^{\frac{\pi}{6}} \frac{\cos x}{1+4\sin^2 x}dx$$

$u = 2\sin x$

$\frac{du}{dx} = 2\cos x$

$du = 2\cos x\, dx$

$\boxed{\frac{1}{2}du = \cos x\, dx}$

Change limits

$u = 2\sin\frac{\pi}{6} = 2\left(\frac{1}{2}\right) = 1$

$u = 2\sin 0 = 0$

$$\frac{1}{2}\int_0^1 \frac{1}{1+u^2}\,du$$

$$\frac{1}{2}\left[\tan^{-1}(u)\right]_0^1 = \frac{1}{2}\tan^{-1}(1) - \frac{1}{2}\tan^{-1}(0)$$

$$= \frac{1}{2}\left(\frac{\pi}{4}\right) - \frac{1}{2}(0)$$

$$= \frac{\pi}{8}$$

Identifying Inverse trig. functions when integrating

2017 Paper: Use the substitution $u = 5x^2$ to find the exact value of

$$\int_0^{\frac{1}{\sqrt{10}}} \frac{x}{\sqrt{1-25x^4}}dx$$

$u = 5x^2$

$\frac{du}{dx} = 10x$

$du = 10x\, dx$

$\boxed{\frac{1}{10}du = x\, dx}$

Change limits

$u = 5\left(\frac{1}{\sqrt{10}}\right)^2 = \frac{1}{2}$

$u = 5(0)^2 = 0$

$$\frac{1}{10}\int_0^{\frac{1}{2}} \frac{1}{\sqrt{1-u^2}}\,du$$

$$= \frac{1}{10}\left[\sin^{-1}(u)\right]_0^{\frac{1}{2}} = \frac{1}{10}\left[\sin^{-1}\left(\frac{1}{2}\right)\right] - \frac{1}{10}\left[\sin^{-1}0\right]$$

$$= \frac{1}{10}\left(\frac{\pi}{6}\right) - 0 = \frac{\pi}{60}$$