

Mixed Integration and differentiation Solutions

Higher

Q1. $\frac{dy}{dx} = 3x^2 + 2x + 1$ $(-1, -1)$

$$\int 3x^2 + 2x + 1 \, dx = \frac{3x^3}{3} + \frac{2x^2}{2} + 1x + c = x^3 + x^2 + x + c = y$$

✓ integration

$$-1 = (-1)^3 + (-1)^2 + (-1) + c$$

✓ substitution

$$-1 = -1 + 1 - 1 + c$$

$$-1 = -1 + c$$

$$0 = c$$

✓ value for c

$$y = x^3 + x^2 + x$$

✓ y in terms of x

$\frac{4}{4}$

Q2. $f(x) = 2x^2(2+x)$ max. and min. on interval $-1 \leq x \leq 1$

$$f'(x) = 0 \text{ (Stationary points)} \quad f(x) = 4x^2 + 2x^3$$

✓ diff + equate to zero

$$f'(x) = 8x + 6x^2 = 0$$

$$\Rightarrow 2x(4 + 3x) = 0$$

$$x = 0 \quad x = -\frac{4}{3}$$

✓ values for x

$$y = 0$$

Not in interval

$$f(-1) = 4(-1)^2 + 2(-1)^3 = 4 - 2 = 2$$

✓

$$f(1) = 4(1)^2 + 2(1)^3 = 4 + 2 = 6$$

✓ substitution

$$\text{min. value} = 0 \quad \text{max. value} = 6$$

✓ state max. + min.

$\frac{6}{6}$

Q3. $y = \frac{\sqrt{x} + 1}{(\sqrt{x})^3}$ $x = 4$ find rate of change

$$\Rightarrow y = \frac{x^{\frac{1}{2}} + 1}{x^{\frac{3}{2}}} = x^{-\frac{3}{2}}(x^{\frac{1}{2}} + 1) = x^{-1} + x^{-\frac{3}{2}}$$

✓ rewrite x

$$\frac{dy}{dx} = -x^{-2} - \frac{3}{2}x^{-\frac{5}{2}} \Rightarrow \frac{-1}{x^2} - \frac{3}{2(\sqrt{x})^5} \quad x = 4$$

✓ differentiate

$$\Rightarrow \frac{-1}{4^2} - \frac{3}{2(\sqrt{4})^5} = \frac{-1}{16} - \frac{3}{64} = \frac{-4}{64} - \frac{3}{64} = \frac{-7}{64}$$

✓ substitute

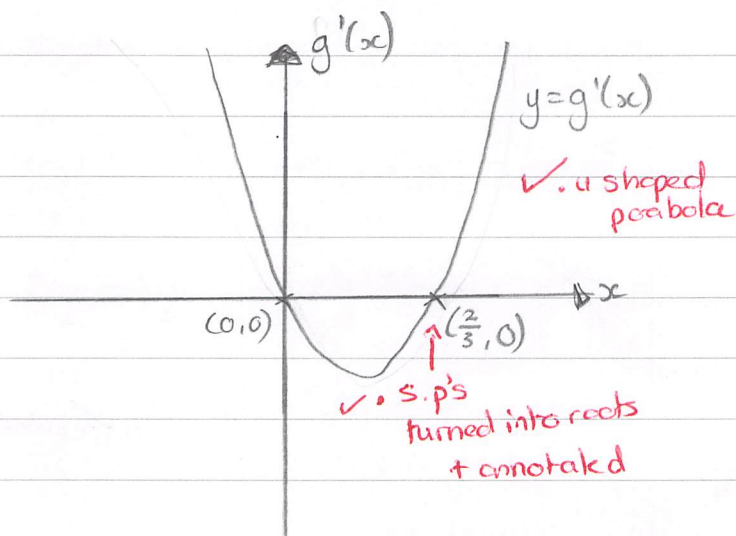
4

✓ answer

$\frac{4}{4}$

Q4. $g(x) = 2(x^3 - x^2) = 2x^3 - 2x^2$
 $g'(x) = 6x^2 - 4x$

$\frac{2}{2}$



Q5. $(x+1)^2(x+4) = y$

Roots $\Rightarrow y=0 \Rightarrow x=-1, x=-4$ $(-1,0)$ $(-4,0)$ ✓ roots

y-intercept $\Rightarrow x=0 \Rightarrow (0+1)^2(0+4) = 4$ $(0,4)$ ✓ y-intercept

$y = (x+1)^2(x+4)$

$y = (x+4)(x^2+2x+1) = x^3 + 2x^2 + x + 4x^2 + 8x + 4$

$= x^3 + 6x^2 + 9x + 4$

S.P.'s

$\frac{dy}{dx} = 3x^2 + 12x + 9 = 0$ ✓ differentiate + equate to zero

$x^2 + 4x + 3 = 0$

$(x+1)(x+3) = 0$

$x = -1$

$x = -3$ ✓ x-values

$y = 0$

$(-1,0)$

$y = 4$

$(-3,4)$

x	-1 ⁻	-1	-1 ⁺	-3 ⁻	-3	-3 ⁺
$\frac{dy}{dx}$	-	0	+	+	0	-
slope	\	-	/	/	-	\

✓ nature table

{Graph on next page}

Q6. $y = x^2 - 5x$ find shaded area

limits

$x^2 - 5x = 0$

$x(x-5) = 0$

$x=0$ $x=5$

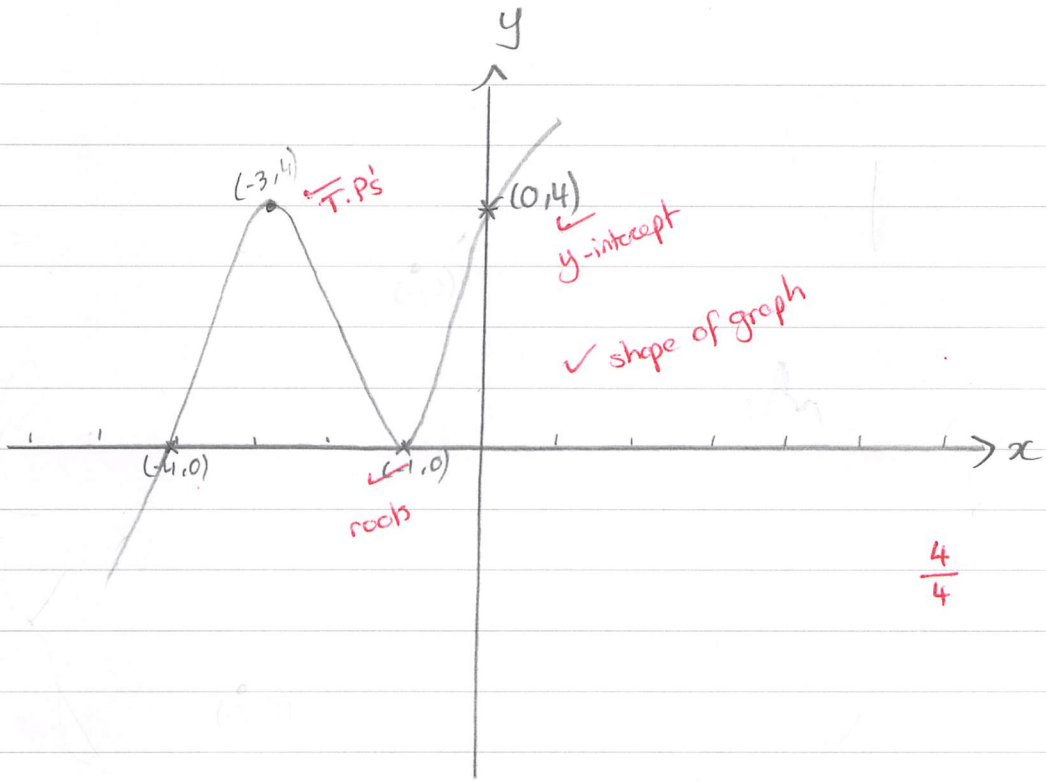
✓ limits

$\int_0^5 x^2 - 5x \, dx = \left[\frac{x^3}{3} - \frac{5x^2}{2} \right]_0^5$
 ✓ ready to integrate
 ✓ integrated
 $\Rightarrow \left[\frac{5^3}{3} - \frac{5(5)^2}{2} \right] - [0]$
 ✓ substitution

$= \frac{125}{3} - \frac{125}{2} = -\frac{125}{6}$

$\frac{5}{5}$

$= \frac{125}{6} \text{ units}^2$
 ✓ Answer (positive)



$$\frac{4}{4}$$

