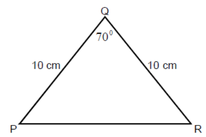
Q1. Calculate the area of PQR



$Area = \frac{1}{2}ab\sin C$

$= 0.5 \times 10 \times 10 \times \sin 70°$

$= 46.98cm^2$

2 marks

A line passes through (-2, 3) and (3, -5). State the equation of the line.

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-5-3}{3-(-2)} = \frac{-8}{5}$

$y - b = m(x - a)$

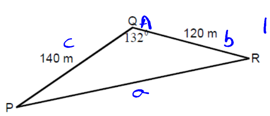
$y - 3 = \frac{-8}{5}(x+2)$

$5y - 15 = -8x - 16$

$8x + 5y + 1 = 0$

3 marks

Q3. Calculate the length of PR



$a^2 = b^2 + c^2 - 2bc\cos A$

$|PR|^2 = 120^2 + 140^2 - (2 \times 120 \times 140 \times \cos 132)$

$= 56\,482.79$

$|PR| = \sqrt{56482.79} = \underline{237.66m}$

3 marks

Find the equation of the line through the point (–1, 4) which is parallel to the line with equation $3x - y + 2 = 0$.

parallel lines ⇒ equal gradients

$3x - y + 2 = 0$

$y = 3x + 2$

$m = 3$

$y - b = m(x - a)$

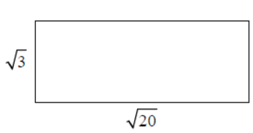
$y - 4 = 3(x+1)$

$y - 4 = 3x + 3$

$3x - y + 7 = 0$

3 marks

Q5. Calculate the area of the rectangle, give your answer as a surd in its simplest form.



$A = L \times B$

$A = \sqrt{3} \times \sqrt{20}$

$A = \sqrt{60}$

$A = \sqrt{4}\sqrt{15}$

$A = 2\sqrt{15}$

3 marks

A function f is given by $f(x) = 2x^2 - x - 9$.

Which of the following describes the nature of the roots of $f(x) = 0$?

A No real roots

B Equal roots

C Real distinct roots

D Rational distinct roots

$b^2 - 4ac$

$a = 2 \quad b = -1 \quad c = -9$

$(-1)^2 - 4(2)(-9)$

$1 + 72$

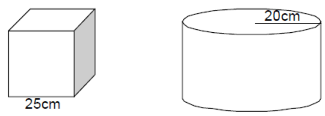
$= 73 > 0$

⇒ Real roots distinct

3 marks

Q7.

In the diagram below the volume of the cylinder is **double** that of the cube.



$V = \pi r^2 h$

Calculate the height of the cylinder.

$V = L \times B \times H$

$V = 25 \times 25 \times 25$

$V = 15625\text{cm}^3$

$V = 15625 \times 2 = 31250\text{cm}^3$

$31250 = \pi \times 20^2 \times h$

$31250 \div (\pi \times 20^2) = h$

$h = 24.87\text{cm}$

4 marks

Find the range of values of k such that the equation $kx^2 - x - 1 = 0$ has no real roots.

no real roots $\Rightarrow b^2 - 4ac < 0$

$a = k \quad b = -1 \quad c = -1$

$(-1)^2 - 4(k)(-1) < 0$

$1 + 4k < 0$

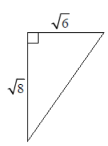
$4k < -1$

$k < -\frac{1}{4}$

3 marks

Q9.

Calculate the area of the triangle, give your answer as a surd in its simplest form.



$A = \frac{1}{2}(L \times B)$

$A = \frac{1}{2}(\sqrt{6} \times \sqrt{8})$

$A = \frac{1}{2}(\sqrt{48})$

$A = \frac{1}{2}(\sqrt{16}\sqrt{3})$

$A = \frac{4\sqrt{3}}{2} = 2\sqrt{3}$

3 marks

Here are two statements about the roots of the equation $x^2 + x + 1 = 0$:

(1) the roots are equal;

(2) the roots are real.

Which of the following is true?

A Neither statement is correct.

B Only statement (1) is correct.

C Only statement (2) is correct.

D Both statements are correct.

$b^2 - 4ac$

$a = 1 \quad b = 1 \quad c = 1$

$1^2 - 4(1)(1)$

$1 - 4 = -3 < 0$

3 marks

Q11. The stem and leaf diagram shows the cost of cars in a show room.

10	4 5 5 7
11	2 3 6
12	0 1 5 5 5 7
13	1 8 9

10 | 4 represents £10 400

(a) Find the range of the costs

(b) Find the median cost.

(a) $= £13900 - 10400$

$= £3500$

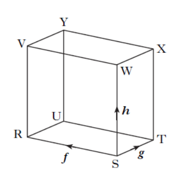
(b) Median $= \frac{12000 + 12100}{2} = £12050$

2 marks

In the diagram RSTU, VWXY represents a cuboid.

$\vec{SR}$ represents vector $\boldsymbol{f}$, $\vec{ST}$ represents vector $\boldsymbol{g}$ and $\vec{SW}$ represents vector $\boldsymbol{h}$.

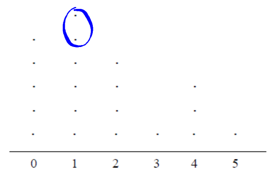
Express $\vec{VT}$ in terms of $\boldsymbol{f}$, $\boldsymbol{g}$ and $\boldsymbol{h}$.



$\vec{VT} = -\underline{h} - \underline{f} + \underline{g}$

1 mark

Q13. The number goals scored by 20 football teams on Saturday were



(a) What is the modal number of goals scored? 1 (most common)

(b) Find the median. 1

2 marks

The equation $3x^2 + x + m = 0$ has equal roots. What is the value of m?

$b^2 - 4ac = 0$

$a = 3 \quad b = 1 \quad c = m$

$(1)^2 - 4(3)(m) = 0$

$1 - 12m = 0$

$1 = 12m$

$m = \frac{1}{12}$

3 marks

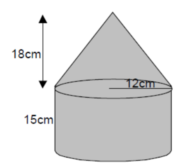
Q15. Calculate the total volume

$V = \pi r^2 h + \frac{1}{3}\pi r^2 h$

$V = (\pi \times 12^2 \times 15) + (\frac{1}{3} \times \pi \times 12^2 \times 18)$

$V = 6785.84 + 2714.34$

$V = 9500.18 cm^3$



5 marks

$7 - 8x - x^2$ is expressed in the form $a - (x + b)^2$. What is the value of a?

$-1[x^2 + 8x - 7]$

$= -1[(x+4)^2 - 16 - 7]$

$= -1[(x+4)^2 - 23]$

$= -(x+4)^2 - 23$

$a = -23$

Higher Level

Q17. Simplify

$$\frac{5n^{-2} \times 4n^5}{10n^{-3}} = \frac{20n^3}{10n^{-3}}$$

$$= 2n^6$$

2 marks

Here are two statements about the roots of equation

$x^2 - x - 2 = 0$

$b^2 - 4ac$

$a = 1 \quad b = -1 \quad c = -2$

$(-1)^2 - 4(1)(-2)$

$1 + 8 = 9 > 0$

Perfect square

(1) The roots are rational

(2) The roots are real

Which of the following is true?

A Neither statement is correct.

B Only statement 1 is correct.

C Only statement 2 is correct.

D Both statments are correct.

3 marks

Q19.

The cost of a holiday increased by 8% from the years 2001 to 2002. If it cost £540 for the holiday in 2002, what was the cost in 2001?

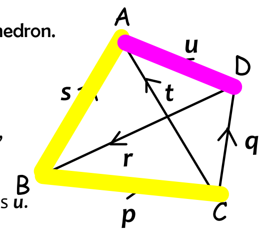
£540 = 108%

1% = 540 ÷ 108 = 5

100% = 5 × 100 = £500

3 marks

In the diagram, ABCD represents a tetrahedron.



$\overrightarrow{BC}$ represents **p**, $\overrightarrow{CD}$ represents **q**,

$\overrightarrow{DB}$ represents **r**, $\overrightarrow{BA}$ represents **s**,

$\overrightarrow{CA}$ represents **t** and $\overrightarrow{DA}$ represents **u**.

One of the statements is false, which one?

A $p = -q + s - u$

B [illegible]

C $r = -p - t + u$

D $s = p + q + u$

1 mark

Q21. Find the value of

$$25^{\frac{3}{2}} = \left(\sqrt{25}\right)^3 = 5^3 = 125$$

2 marks

A line through the points A(2k, 3) and B(k, 5) has a gradient of 4. What is the value of k?

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{5-3}{k-2k} = 4$$

$$\frac{2}{-1k} = 4$$

$$2 = -4k$$

$$k = -\tfrac{1}{2}$$

3 marks

The marks of 7 pupils in an advanced higher maths exam were

77 67 43 90 66 93 75

Q23.

Calculate the mean and standard deviation of these marks.

Another group of 7 pupils who sat the same exam had a mean of 78 and a standard deviation of 3.2.
Make two comparisons of the marks of the two groups.

$\bar{x} = 511 \div 7 = 73$

x	$x - \bar{x}$	$(x-\bar{x})^2$
77	4	16
67	-6	36
43	-30	900
90	17	289
66	-7	49
93	20	400
75	2	4
$n=7$		1694

$s = \sqrt{\frac{\Sigma(x-\bar{x})^2}{n-1}}$

$s = \sqrt{\frac{1694}{7-1}}$

$s = \sqrt{282.33}$

$s = 16.8$

the other group of students on average performed better (higher mean) and got more consistent results (lower standard deviation).

4 + 2 marks

(x + 4)(x - 2) can be written in the form $(x + a)^2 + b$. What is the value of b?

$x^2 - 2x + 4x - 8$

$x^2 + 2x - 8$

$(x+1)^2 - 1 - 8$

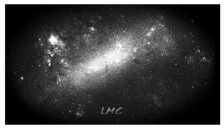
$(x+1)^2 - 9$

$b = -9$

2 marks

Q25.

The large Magellanic cloud is 1.69×10^{18} kilometres from Earth. Write this distance as an ordinary number.



1690000000 000 000 000

1 mark

Q26. Solve

$$\frac{x(x+5)}{4} = 9$$

$\frac{x^2+5x}{4} = 9$

$\times 4$ $\times 4$

$x^2+5x = 36$

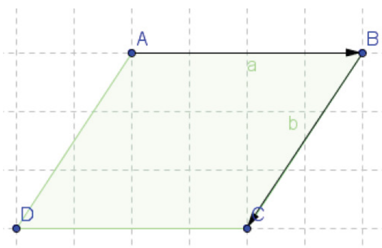
$x^2+5x-36 = 0$

$(x - 4)(x + 9) = 0$

$x = 4$ or $x = -9$

4 marks

Q27. Write the vector AC in terms of A and B



$\vec{AC} = \underline{a} + \underline{b}$

1 mark

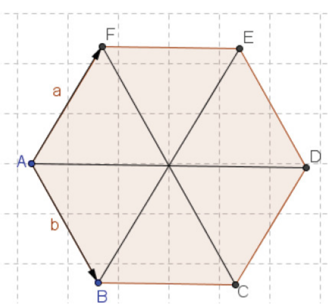
Q28.

$(t^4)^3 = t^{12}$

1 mark

Write the vector AE in terms of a and b

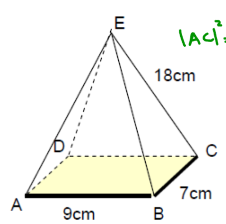
Q29.



$\vec{AE} = \underline{b} + 2\underline{a}$

1 mark

Q30. Find the diagonal AC

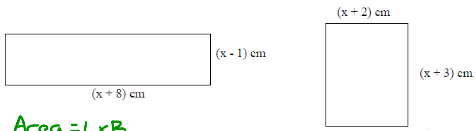


$|AC|^2 = 9^2 + 7^2 = 130$

$= \sqrt{130} = 11.4cm$

2 marks

The following rectangles have the same area. Find the value of x.



Area = L×B

⇒ Area = $(x+8)(x-1)$

Area = $(x+2)(x+3)$

So $(x+8)(x-1) = (x+2)(x+3)$ ✓

$x^2 - 1x + 8x - 8 = x^2 + 3x + 2x + 6$

$x^2 + 7x - 8 = x^2 + 5x + 6$ ✓

$-x^2 \qquad -x^2$

$7x - 8 = 5x + 6$ ✓

$+8 \qquad +8$

$7x = 5x + 14$

$-5x \qquad -5x$

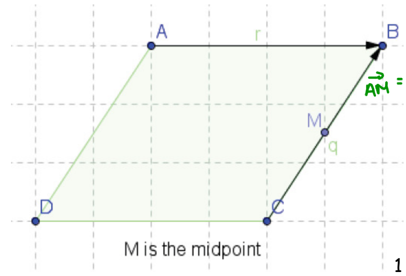
$2x = 14$

$\div 2 \qquad \div 2$

$x = 7$ ✓

4 marks

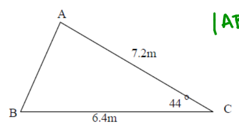
Q32.
Write the vector AM in terms of r and q



$\vec{AM} = r - \frac{1}{2}q$ ✓

1 mark

5. The following diagrams show a triangle ABC. *Marks*



(a) Calculate the length of AB (to 2 significant figures). **Do not use a scale drawing.** 4

(b) Calculate the area of triangle ABC. 2

(a) $a^2 = b^2 + c^2 - 2bc\cos A$

$|AB|^2 = 6.4^2 + 7.2^2 - (2 \times 6.4 \times 7.2 \times \cos 44°)$ ✓

$|AB|^2 = 26.51$ ✓

$AB = \sqrt{26.51}$ ✓

$\doteq 5.15m$

$\rightarrow 5.2m$ ✓

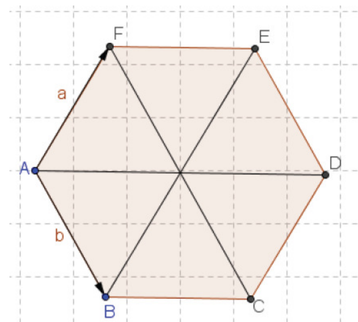
Area = $\frac{1}{2}ab\sin C$

$= \frac{1}{2} \times 6.4 \times 7.2 \times \sin 44$ ✓

$= 16m^2$ ✓

Write the vector AD in terms of a and b

Q34.



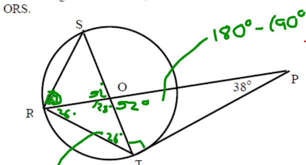
$\vec{AD} = 2a + 2b$

or

$2(a + b)$ ✓

1 mark

10. In the diagram below PT is a tangent to the circle, O is the center of the circle and angle RPT is 38°. Find the size of angle ORS.



$180° - (90° + 38°)$ ✓

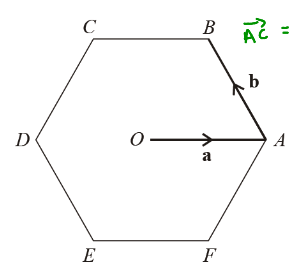
$(180° - 128°) \div 2$ ✓

3

$O\hat{R}S = 90° - 26° = 64°$ ✓

Write the vector AC in terms of a and b

Q36.



$\vec{AC} = b - a$ ✓

1 mark

Q37.

Evaluate $3\frac{2}{5} - 1\frac{3}{4}$

$\frac{17}{5}^{\times 4} - \frac{7}{4}^{\times 5}$ LCM = 20

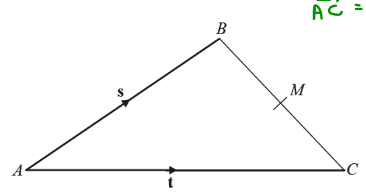
$= \frac{68}{20} - \frac{35}{20} = \frac{33}{20} = 1\frac{13}{20}$

2 marks

Write the vector BC in terms of s and t

Q38.

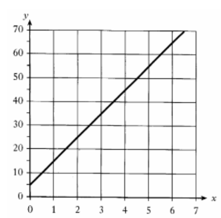
$\vec{AC} = \frac{1}{2}(s+t)$



1 mark

Q39.

$(0,5)$ $(2,25)$



$m = \frac{25-5}{2-0} = \frac{20}{2} = 10$

$c = 5$

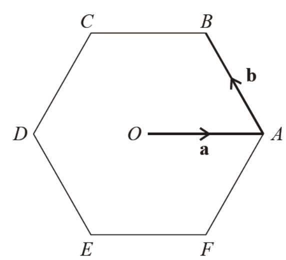
$y = 10x + 5$

Find the equation of this straight line in the form $y = mx + c$

3 marks

Write the vector EC in terms of s and t

Q40.



$\vec{EC} = b + a + b$
$= a + 2b$

1 mark

Q41.

Express $y = x^2 + 8x - 7$ in the form $y = (x+a)^2 + b$ and hence state the coordinates of the turning point.

$y = (x+4)^2 - 16 - 7$

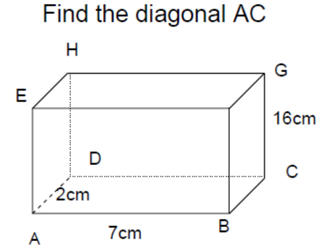
$y = (x+4)^2 - 23$

$TP = (-4, -23)$

3 marks

Q42.

Find the diagonal AC



$AC^2 = 7^2 + 2^2$

$AC^2 = 53$

$AC = \sqrt{53}$

$= 7.3$ cm

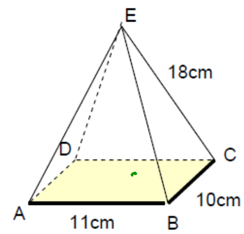
2 marks

Q43.

$$t^{11} \div t^5 \quad = \frac{t^{11}}{t^5} = t^6$$

1 mark

Q44. Find the height



$11^2 + 10^2 = 221$

$AC = \sqrt{221} = 14.9cm$

$\frac{1}{2}AC = 7.45cm$

$18^2 - 7.5^2 = 267.75$

$height = \sqrt{267.75}$

$= 16.4cm$

4 marks

Q45.

$$\frac{y^7}{y^2} = y^5$$

1 mark

Q46. Rationalise the denominator

$$\frac{12}{\sqrt{6}} \begin{matrix} \times\sqrt{6} \\ \times\sqrt{6} \end{matrix}$$

$= \frac{12\sqrt{6}}{6} = 2\sqrt{6}$

2 marks

Q47. Simplify

$$(3x^2y)^2 = 9x^4y^2$$

2 marks

Q48. Multiply out and simplify

$$(1 + \sqrt{2})^2$$

$(1+\sqrt{2})(1+\sqrt{2})$

$= 1 + \sqrt{2} + \sqrt{2} + \sqrt{4}$

$= 1 + 2\sqrt{2} + 2$

$= 3 + 2\sqrt{2}$

2 marks

Q49.

$$\frac{6y^5 \times 2y^6}{4y^8}$$

$= \frac{12y^{11}}{4y^8} = 3y^3$

2 marks

Q50.

$$\frac{8y^9}{2y \times 2y^3}$$

$= \frac{8y^9}{4y^4} = 2y^5$

2 marks

Q51.

Solve

$$4x - 5 > 2x - 15$$

$4x - 5 > 2x - 15$
$+5 \qquad +5$
$4x > 2x - 10$
$-2x \quad -2x$
$2x > -10$
$x > -5$

3 marks

Q52.

$$\sqrt{20} + \sqrt{45}$$

$\sqrt{4}\sqrt{5} + \sqrt{9}\sqrt{5}$
$= 2\sqrt{5} + 3\sqrt{5} = 5\sqrt{5}$

3 marks

Q53.

$P = R^3 b - 5$

Change the subject of the formula to R.

$P = R^3 b - 5$
$P + 5 = R^3 b$
$R^3 = \frac{P+5}{b}$
$R = \sqrt[3]{\frac{P+5}{b}}$

3 marks

Q54.

$$\sqrt{75} - \sqrt{48}$$

$\sqrt{25}\sqrt{3} - \sqrt{16}\sqrt{3}$
$= 5\sqrt{3} - 4\sqrt{3} = 1\sqrt{3}$

3 marks

Q55.

Two vectors are defined as $\boldsymbol{u} = \begin{pmatrix} 2 \\ -5 \end{pmatrix}$ and $\boldsymbol{v} = \begin{pmatrix} -4 \\ 3 \end{pmatrix}$.

(a) Find the resultant vector $\boldsymbol{u} + 3\boldsymbol{v}$.

$3v = \begin{pmatrix} -12 \\ 9 \end{pmatrix}$

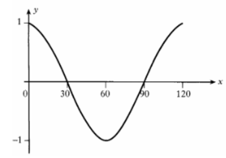
$u + 3v = \begin{pmatrix} 2 \\ -5 \end{pmatrix} + \begin{pmatrix} -12 \\ 9 \end{pmatrix} = \begin{pmatrix} -10 \\ 4 \end{pmatrix}$

(b) Find $|\boldsymbol{u} + 3\boldsymbol{v}|$.

$|u+3v| = \sqrt{(-10)^2 + 4^2} = \sqrt{116} = 2\sqrt{29}$

4 marks

Q56.



$b = 3$

Part of the graph of $y = \cos bx°$ is shown in the diagram.

State the value of b.

1 mark

Q57.

Find the point of intersection of the straight lines with equations

$2x + y = 5$ and $x - 3y = 6$.

$2x + y = 5 \quad \times 3$
$x - 3y = 6$

$6x + 3y = 15$
$x - 3y = 6$
$7x = 21$
$x = 3$

$3 - 3y = 6$
$-3y = 3$
$y = -1$

$(3, -1)$

3 marks

Q58.

$b^2 - 4ac \qquad a = 1 \quad b = -3 \quad c = 7$

A parabola has equation $y = x^2 - 3x + 7$.

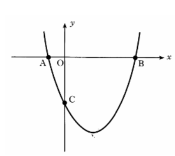
$(-3)^2 - 4(1)(7)$
$9 - 28$
$= -19$

Using the discriminant, determine the nature of its roots.

$b^2 - 4ac < 0$
$\Rightarrow$ Roots not real

3 marks

Q59.



The equation of the parabola in the diagram above is $y = (x-2)^2 - 9$.

(a) State the coordinates of the minimum turning point of the parabola. $TP = (2, -9)$

(b) Find the coordinates of C. y-intercept $\Rightarrow x = 0$

$y = (0-2)^2 - 9$
$y = 4 - 9 = -5$
$(0, -5)$

(c) A is the point $(-1, 0)$. State the coordinates of B.

$A(-1, 0) \qquad B(5, 0)$ as axis of symmetry $\Rightarrow x = 2$

4 marks

Q60.

$LCM = x(x+2)$

Express $\dfrac{3}{x} - \dfrac{5}{x+2}$, $x \neq 0, x \neq 2$, as a single fraction in its simplest form.

$\dfrac{3(x+2) - 5x}{x(x+2)} = \dfrac{3x + 6 - 5x}{x(x+2)} = \dfrac{-2x + 6}{x(x+2)}$

3 marks

Q61.

The total emissions of greenhouse gases by the USA in 2007 amounted to the equivalent of 7.2million tonnes of carbon dioxide. If the annual increase in emissions is 1.2%, calculate the total amount of emissions of greenhouse gases by the USA expected in 2010. Give your answer in millions of tonnes to 2 s.f.

$7.2 \times 1.012^3 = 7.462322842$

$\rightarrow$ 7.5 million tonnes

4 marks

Q62. Multiply out and simplify

(3x - 1)(2x² + 3x - 4)

$6x^3 + 9x^2 - 12x - 2x^2 - 3x + 4$

$6x^3 + 7x^2 - 15x + 4$

3 marks

Q63.

Change the subject of the formula to r.

$$A = 4\pi r^2.$$

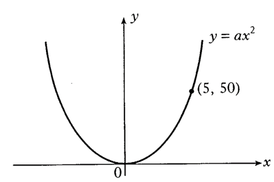
$\frac{A}{4\pi} = r^2$

$r = \sqrt{\frac{A}{4\pi}}$

2 marks

Q64.

The diagram below shows the graph of $y = ax^2$.



$50 = a(5)^2$

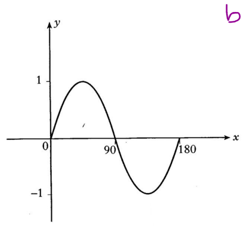
$50 = 25a$

$a = 2$

Find the value of a. 2 marks 2

Q65.

$b = 2$



The graph of $y = \sin bx°$ is shown in the diagram.
State the value of b. 1

1 mark

Q66.

150 patients have been given a flu vaccine.
The data is shown in the table below.

AGE	*GENDER*	
	male	*female*
5 or under	4	3
6 – 15	7	8
16 – 59	37	47
60 or over	12	32

150

What is the probability that

(*a*) a patient given the flu vaccine was male **and** aged 60 or over? $\frac{12}{150} = \frac{2}{25}$

(*b*) a patient given the flu vaccine was aged 5 or under?

$4 + 3 = \frac{7}{150}$

2 marks

Q67.

Joan buys gold and silver charms to make bracelets.

2 gold charms and 5 silver charms cost £125.

(*a*) Let *g* pounds be the cost of one gold charm and *s* pounds be the cost of one silver charm.

Write down an equation in terms of *g* and *s* to illustrate the above information.

$2g + 5s = 125$

4 gold charms and 3 silver charms cost £145. $4g + 3s = 145$

(*b*) Write down another equation in terms of *g* and *s* to illustrate this information.

(*c*) Hence calculate the cost of each type of charm.

$2g + 5s = 125 \quad (\times -2)$
$4g + 3s = 145$

$-4g - 10s = -250$
$4g + 3s = 145$
$-7s = -105$
$s = £15$

$2g + 5(15) = 125$
$2g + 75 = 125$
$2g = 50$
$g = £25$

A charm costs £15

A chain costs £25

6 marks

Q68.

Solve the inequality

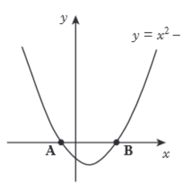
$$4x - 5 \leq 7x - 20.$$

$+5 \qquad +5$
$4x \leq 7x - 15$
$-7x \qquad -7x$
$-3x \leq -15$
$\div -3 \qquad \div -3$
$x \geq 5$

3 marks

Q69.

The parabola with equation $y = x^2 - 2x - 3$ cuts the x-axis at the points A and B as shown in the diagram.



$x^2 - 2x - 3 = 0$
$(x+1)(x-3) = 0$
$x + 1 = 0 \qquad x - 3 = 0$
$x = -1 \qquad x = 3$

(*a*) Find the coordinates of A and B. $A(-1, 0)$ $B(3, 0)$ 3 marks

(*b*) Write down the equation of the axis of symmetry of $y = x^2 - 2x - 3$.

$x = 1$

1 mark

Q70.

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Solve the equation

$$2x^2 + 7x - 3 = 0.$$

Give your answers **correct to 1 decimal place**.

$a = 2 \quad b = 7 \quad c = -3$

$$\frac{-7 \pm \sqrt{7^2 - 4 \times 2 \times (-3)}}{2 \times 2}$$

$$= \frac{-7 \pm \sqrt{49 + 24}}{4}$$

$$= \frac{-7 \pm \sqrt{73}}{4}$$

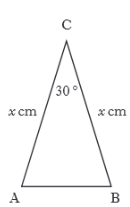
$x = 0.4$ or $x = -3.9$

4 marks

Q71.

ABC is an isosceles triangle with angle ACB = 30°.

AC = BC = x centimetres.



Area $= \frac{1}{2}ab\sin C$
$= \frac{1}{2} x \times x \times \sin 30$
$9 = \frac{1}{2} x^2 \times \sin 30$

The area of triangle ABC is 9 square centimetres.

Calculate the value of x.

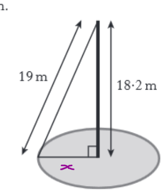
$18 = x^2 \times \sin 30$
$18 = x^2 \times \frac{1}{2}$
$36 = x^2$
$x = \sqrt{36} = 6\text{cm}$

3 marks

Q72.

A mobile phone mast, 18·2 metres high, stands vertically in the centre of a circle.

It is supported by a wire rope, 19 metres long, attached to the ground at a point on the circumference of the circle, as shown.



$19^2 - 18.2^2 = 29.76$
$x^2 = 29.76$
$x = 5.5\text{m}$

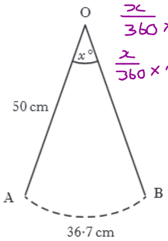
$C = \pi \times D$
$C = \pi \times 11 = 34.6\text{m}$

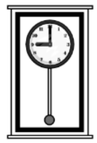
Calculate the circumference of the circle.

3 marks

Q73.

As the pendulum of a clock swings, its tip moves through an arc of a circle.





$\frac{x}{360} \times \pi D$

$\frac{x}{360} \times \pi \times 100$

$= 36.7$ ✓

$\frac{x}{360} \times 314.2 = 36.7$ ✓

$\frac{x}{360} = 0.1168...$ ✓

$x = 42°$ ✓

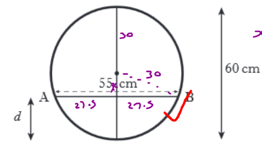
The length of the pendulum is 50 centimetres.
The length of the arc is 36·7 centimetres.

Calculate $x°$, the angle through which the pendulum swings.

4 marks

Q74.

Water flows through a horizontal pipe of diameter 60 centimetres.
The surface width, AB, of the water is 55 centimetres.



$x^2 = 30^2 - 27.5^2$ ✓

$x^2 = 143.75$

$x = \sqrt{143.75}$

$x = 11.99 \to 12$ ✓

$d = 30 - 12 = 18\text{cm}$ ✓

(*a*) Calculate the depth, *d*, of the water in the pipe.

(*b*) What other depth of water would give the same surface width? ✓

$60 - 18 = 42\text{cm}$ ✓

5 marks

Q75.

(*a*) Express $\frac{a^{\frac{1}{2}} \times a^{\frac{5}{2}}}{a^2}$ in its simplest form. $\frac{a^{\frac{6}{2}}}{a^2} = \frac{a^3}{a^2} = a$ ✓ ✓

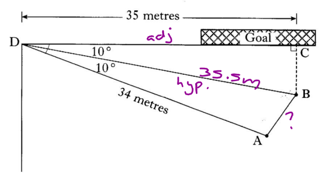
(*b*) Express $\frac{2}{\sqrt{3}}$ as a fraction with a rational denominator. $\frac{2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$ ✓ ✓

(*c*) Express $\frac{2}{x} + \frac{4}{x+3}$, $x \neq 0$, $x \neq -3$, as a single fraction in its simplest form.

$\frac{2(x+3) + 4x}{x(x+3)} = \frac{2x + 6 + 4x}{x(x+3)} = \frac{6x+6}{x(x+3)}$ ✓ ✓ ✓

7 marks

Q76.



DB first

$\cos 10° = \frac{35}{DB}$ ✓

$DB = \frac{35}{\cos 10°}$

$DB = 35.5m$ ✓

The diagram shows part of a football pitch with players at A, B, C and D.
BC is perpendicular to CD.
CD = 35 metres, angle CDB = 10°, angle BDA = 10°, AD = 34 metres.
Find the distance from A to B. **5**

$a^2 = b^2 + c^2 - 2bc\cos A$

$AB^2 = 34^2 + 35.5^2 - 2 \times 34 \times 35.5 \times \cos 10°$ ✓

$AB^2 = 38.92$ ✓

$AB = \sqrt{38.92} = 6.3m$ ✓

Q77.

Change the subject of the formula $r = \frac{st}{q}$ to s.

$qr = st$ ✓

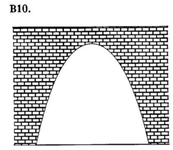
$\div t \quad \div t$

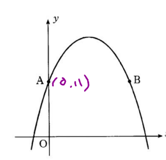
$\frac{qr}{t} = s$ ✓

2 marks

Q78.

B10.





The arch of a railway bridge is represented by a parabola. The equation of the parabola is

$$y = 20 - (x - 3)^2.$$

(*a*) State the coordinates of the maximum turning point of the parabola. **2**

$(3, 20)$ ✓ ✓

(*b*) State the equation of the axis of symmetry. **1**

$x = 3$ ✓

(*c*) Points A and B have the same *y*-coordinate.
A is the point (0, 11). State the coordinates of B. **2**

$0 + 3 + 3 = 6$ ✓

$B(6, 11)$ ✓

Q79.

Solve the equation

$4\sin x° - 1 = 0, \qquad 0 \le x < 360.$ 3

$+1 \quad +1$

$4\sin x = 1$

$\div 4 \quad \div 4$

$\sin x = 0{\cdot}25$

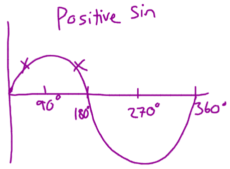
$x = \sin^{-1} 0{\cdot}25$

$x = 14{\cdot}5°$

$x = 180° - 14{\cdot}5°$

$= 165{\cdot}5°$

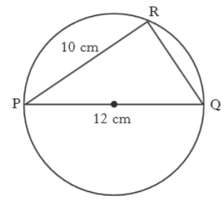
Positive sin



Q80.

In the diagram,

- PQ is the diameter of the circle
- PQ = 12 centimetres
- PR = 10 centimetres.



Calculate the length of QR.

Give your answer as a surd in its simplest form.

$QR^2 = 12^2 - 10^2$

$= 144 - 100$

$= 44$

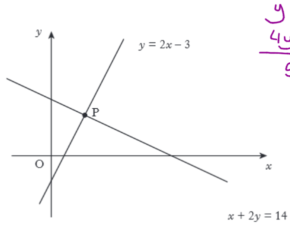
$QR = \sqrt{44} = \sqrt{11}\sqrt{4} = 2\sqrt{11}$

3 marks

Q81.

The graph below shows two straight lines.

- $y = 2x - 3$
- $x + 2y = 14$



The lines intersect at the point P.

Find, **algebraically**, the coordinates of P.

$y - 2x = -3$

$2y + x = 14 \quad \times 2$

$y - 2x = -3$

$4y + 2x = 28$

$5y = 25$

$y = 5$

$5 - 2x = -3$

$-2x = -8$

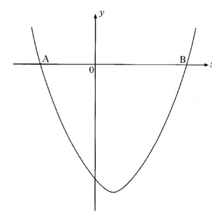
$x = 4$

$P = (4, 5)$

4 marks

Q82.

6.



The equation of the parabola in the above diagram is

$y = (x-1)^2 - 16.$

(a) State the coordinates of the minimum turning point of the parabola. 2

(b) State the equation of the axis of symmetry of the parabola. 1

(c) The parabola cuts the x-axis at A and B. Find the length of AB. 3

$(1, -16)$

$x = 1$

$0 = (x-1)^2 - 16$

$0 = x^2 - 2x + 1 - 16$

$x^2 - 2x - 15 = 0$

$(x+3)(x-5) = 0$

$x + 3 = 0 \qquad x - 5 = 0$

$x = -3 \qquad x = 5$

$(-3, 0) \qquad (5, 0)$

length AB = 8

Q83.

(a) Evaluate $(2^3)^2$. $= 8^2 = 64$

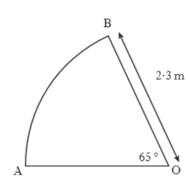
(b) Hence find n, when $(2^3)^n = \frac{1}{64}$. $n = -2$

$\frac{1}{64} = \frac{1}{(2^3)^2} = (2^3)^{-2}$

3 marks

Q84.

A sector of a circle, centre O, is shown below.



The radius of the circle is 2·3 metres.

Angle AOB is 65°.

Find the length of the arc AB.

$\frac{x}{360} \times \pi D$

$\frac{65}{360} \times \pi \times 4{\cdot}6 = 2{\cdot}61\text{m}$

3 marks

Q85.

(*a*) Express $\sqrt{45} - 2\sqrt{5}$ as a surd in its simplest form.

$\sqrt{9}\sqrt{5} - 2\sqrt{5}$
$= 3\sqrt{5} - 2\sqrt{5} = \sqrt{5}$

(*b*) Express as a fraction in its simplest form

$$\frac{1}{x^2} + \frac{1}{x}, \quad x \neq 0.$$

LCM $= x^2$

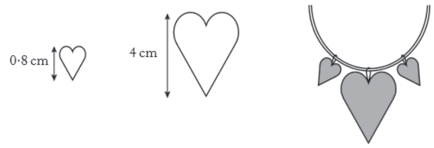
$\Rightarrow \frac{1}{x^2} + \frac{x}{x^2}$

$= \frac{1+x}{x^2}$

4 marks

Q86.

A necklace is made of beads which are mathematically similar.



The height of the smaller bead is 0·8 centimetres and its area is 0·6 square centimetres.

The height of the larger bead is 4 centimetres.

Find the area of the larger bead.

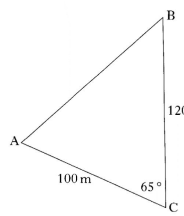
S.f : $4 \div 0.8 = 5$ Area sf $= 5^2 = 25$

$0.6 \times 25 = 15\text{cm}^2$

3 marks

Q87.

1. The sketch shows a triangle, ABC.



Calculate the area of the triangle. 2

$\frac{1}{2}ab\sin C$

$\frac{1}{2}\, 100 \times 120 \times \sin 65$

$= 5437.85\text{m}^2$

Q88.

(*a*) (i) Factorise completely

$3y^2 - 6y$. $3y(y-2)$

(ii) Factorise

$y^2 + y - 6$. $(y+3)(y-2)$

(*b*) Hence express $\dfrac{3y^2 - 6y}{y^2 + y - 6}$ in its simplest form.

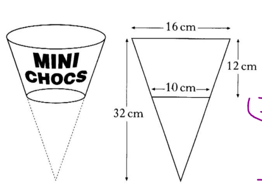
$\dfrac{3y\cancel{(y-2)}}{(y+3)\cancel{(y-2)}}$

$= 3y/y+3$

6 marks

Q89.

6. A container to hold chocolates is in the shape of part of a cone with dimensions as shown below.



Calculate the volume of the container.
Give your answer correct to one significant figure. 5

$\frac{1}{3}\pi r^2 h - \frac{1}{3}\pi r^2 h$

$(\frac{1}{3} \times \pi \times 8^2 \times 32)$

$- (\frac{1}{3} \times \pi \times 5^2 \times 20)$

$= 2144.66$

$- 523.6$

$= 1621.06\text{cm}^3$

$\rightarrow 2000\text{cm}^3$

Q90.

(*a*) Factorise

$x^2 - 4y^2$. $(x+2y)(x-2y)$

(*b*) Expand and simplify

$(2x-1)(x+4)$.

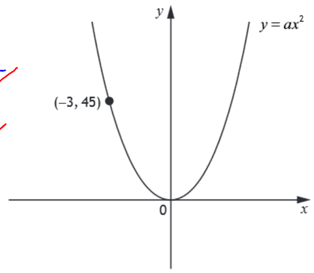
$2x^2 + 8x - x - 4 = 2x^2 + 7x - 4$

(*c*) Expand

$x^{\frac{1}{2}}(3x + x^{-2})$. $= 3x^{\frac{3}{2}} + x^{-\frac{3}{2}}$

5 marks

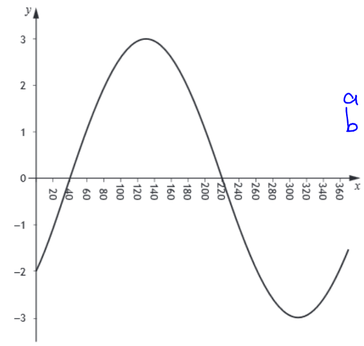
Q91. The diagram below shows part of the graph of $y = ax^2$



$45 = a(-3)^2$
$9a = 45$
$\underline{a = 5}$

Find the value of a. 2

Q92. The graph of $y = a\sin(x+b)°$, $0 \le x \le 360$, is shown below. MARKS

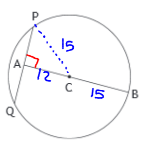


$a = 3$
$b = 40°$
(moved to right by 40°)

Write down the values of a and b. 2

Q93. MARKS

The diagram below shows a circle, centre C.



$AC = 27 - 15$
$= 12cm$

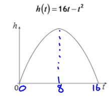
The radius of the circle is 15 centimetres.
A is the mid-point of chord PQ.
The length of AB is 27 centimetres.
Calculate the length of PQ. 4

$(PA)^2 = 15^2 - 12^2$
$= 225 - 144$
$= 81$
$PA = \sqrt{81} = 9cm$
$PQ = 2 \times 9 = \underline{18cm}$

Q94. MARKS

The diagram below shows the path of a small rocket which is fired into the air. The height, h metres, of the rocket after t seconds is given by

$h(t) = 16t - t^2$



(a) After how many seconds will the rocket first be at a height of 60 metres? 4

$\Rightarrow h(t) = 60$
$60 = 16t - t^2$
$t^2 - 16t + 60 = 0$
$(t-6)(t-10) = 0$
$t = 6 \quad , t = 10$
It will first reach 60metres after 6 seconds

(b) Will the rocket reach a height of 70 metres?
Justify your answer. 3

$h(t) = 16t - t^2$
Roots: $16t - t^2 = 0$
$t^2 - 16t = 0$
$t(t - 16) = 0$
$t = 0 \quad t = 16$
max. height when $t = 8$
$16(8) - 8^2$
$= 128 - 64 = 64$metres
$\Rightarrow$ It won't ever reach 70metres.

Q95.

Two groups of people go to a theatre.
Bill buys tickets for 5 adults and 3 children.
The total cost of his tickets is £158·25.

(a) Write down an equation to illustrate this information. 1

$5a + 3c = 158{\cdot}25$

(b) Ben buys tickets for 3 adults and 2 children.
The total cost of his tickets is £98.
Write down an equation to illustrate this information. 1

$3a + 2c = 98$

(c) Calculate the cost of a ticket for an adult and the cost of a ticket for a child. 4

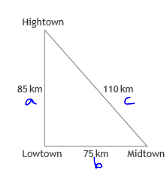
$5a + 3c = 158{\cdot}25 \quad (\times 2) \qquad 10a + 6c = 316{\cdot}5$
$3a + 2c = 98 \qquad (\times 3) \quad \ominus\; 9a + 6c = 294$
$a \qquad = 22{\cdot}50$

$5(22{\cdot}5) + 3c = 158{\cdot}25$
$112{\cdot}5 + 3c = 158{\cdot}25$
$3c = 45{\cdot}75$
$c = 15{\cdot}25$

An adult ticket costs £22.50
A child's ticket costs £15.25

Q96. The diagram below shows the position of three towns.
Lowtown is due west of Midtown.
The distance from

- Lowtown to Midtown is 75 kilometres.
- Midtown to Hightown is 110 kilometres.
- Hightown to Lowtown is 85 kilometres.



Is Hightown directly north of Lowtown?
Justify your answer. 4

Pythagoras states that a triangle is right-angled if $a^2 + b^2 = c^2$ where c is the hypotenuse

$a^2 + b^2 \qquad c^2$
$85^2 + 75^2 \qquad 110^2$
$12850 \neq 12100$

$\Rightarrow$ Hightown is not directly north of Lowtown as triangle is not right-angled.