

Expressions and formula

Revision

Mathematics



National 4

1.1 Applying algebraic skills to manipulating expressions and working with formulae

Using the distributive law in an expression with a numerical common factor to produce a sum of terms.

1) Expand the brackets:

a) $2(3x-3)$

b) $4(5y+2)$

c) $6(2c-1)$

d) $5(4a-5)$

2) Expand the brackets:

a) $3(y+2)$

b) $3(a-2)$

c) $7(n-1)$

d) $2(5+y)$

Factorising a sum of terms with a numerical common factor.

1) Factorise:

a) $3a+6$

b) $5y+10$

c) $7r-7$

d) $81-45y$

Simplifying an expression which has more than one variable.

1) a) $8x + 3y - 4x - y$

b) $3p + q + 2q + p + p + q$

c) $8g + 2h - 7g - h + 3g + 4h$

d) $4x + 4y + 4 - 3x - 3y + 3$

Evaluating an expression or a formula which has more than one variable.

1) a) When $x = 5$ and $y = 3$, find the value of $6x - 2y$.

b) When $a=2$ and $b=3$, find the value of $3a+b$

c) When $c=5$ and $d=6$ find the value of $6c - 2d$

d) When $e=4$ and $f=3$ and $g=2$, find the value of $ef + g$.

2) a) The sum, S of the angles in a polygon with n sides is given by the formula $S=180(n-2)^\circ$. Find S when:

i) $n=7$

ii) $n=12$

b) A magician charges £25 for every show he performs, plus an extra £10 per hour spent on stage. The formula for calculating his charge is $C=10t+25$, where C is the charge in pounds and t is the length of the show in hours. How much does he charge for a show lasting the following durations:

i) 3 hours

ii) 2 hours

c) The surface area of the cuboid is given by the formula:

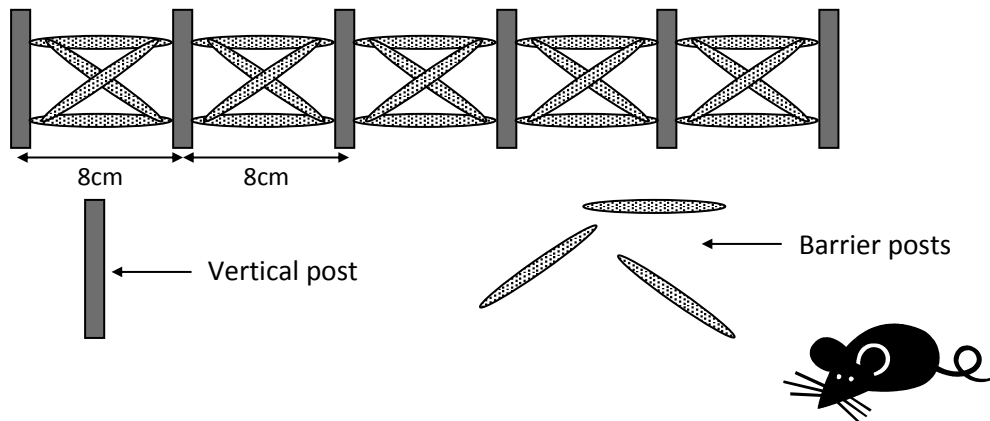
$$S=2ab+2bc+2ac.$$

Find the surface area when $a=3$, $b=4$ and $c=5$.

d) Mass, m can be calculated by dividing the force in Newton's, f by the acceleration, a . Calculate the mass if $f=420$ and $a=70$.

Extending a straightforward number or diagrammatic pattern and determining its formula.

- 1) Rachel plans to build a pen for her pet mouse. She decides to create a fence using counting rods as her vertical posts and lollipop sticks as her barrier posts, as shown below.



- a) Complete the table below.

| | | | | |
|----------------------------------|---|---|---|---|
| Number of vertical posts (v) | 2 | 3 | 4 | 5 |
| Number of barrier posts (b) | | 8 | | |

- b) Write down a formula for calculating the number of barrier posts (b), when you know the number of vertical posts (v).
- c) Each section of fencing is 8cm long, as shown in the diagram above, and Rachel reckons she will need a fence 240cm long.
- How many vertical posts will she need?
 - How many barrier posts will she need?

- 2) Art students at college were asked to design a bracelet.

Julie made up this design from bars and chains

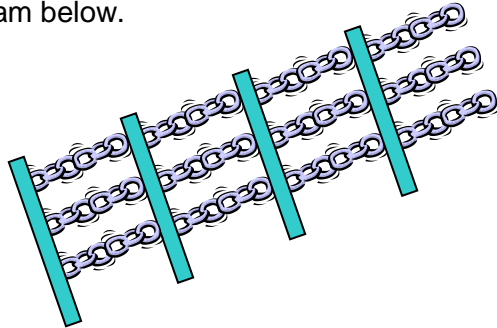


a) Complete this table for the above pattern

| | | | | | |
|--------------------------|---|---|---|--|---|
| Number of bars (b) | 2 | 3 | 4 | | 8 |
| Number of chains (c) | | | | | |

- b) Write down a formula for calculating the number of chains (c) when you know the number of bars (b).
- c) Julie has 50 pieces of chain. How many bars will she need if she wants to use as many of the pieces of chain as possible?

3) A bar gate bracelet is designed with 3 chain links between 2 solid bars as shown in the diagram below.



a) Complete the table below.

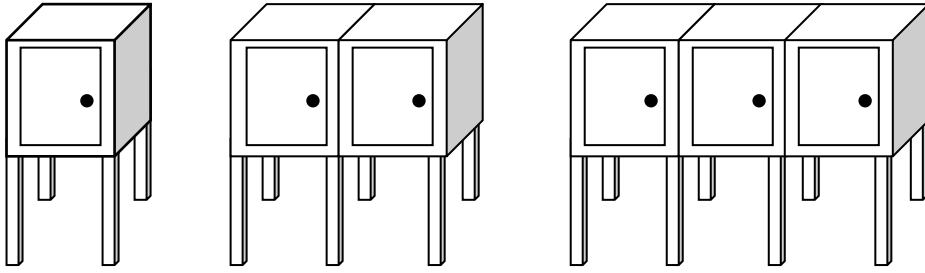
| | | | | | | | |
|--------------------------|---|---|---|---|---|--|----|
| Number of bars (b) | 2 | 3 | 4 | 5 | 6 | | 10 |
| Number of chains (c) | 3 | 6 | 9 | | | | |

b) Find a formula for calculating the number of chain links (c) when you know the number of bars (b).

c) How many bars would be in a bracelet which had 63 gold chain links?

Working must be shown.

4) Elevated storage units come in 1, 2, 3, 4, box sizes, as shown below:



a) Use the diagrams above to help you to complete the following table

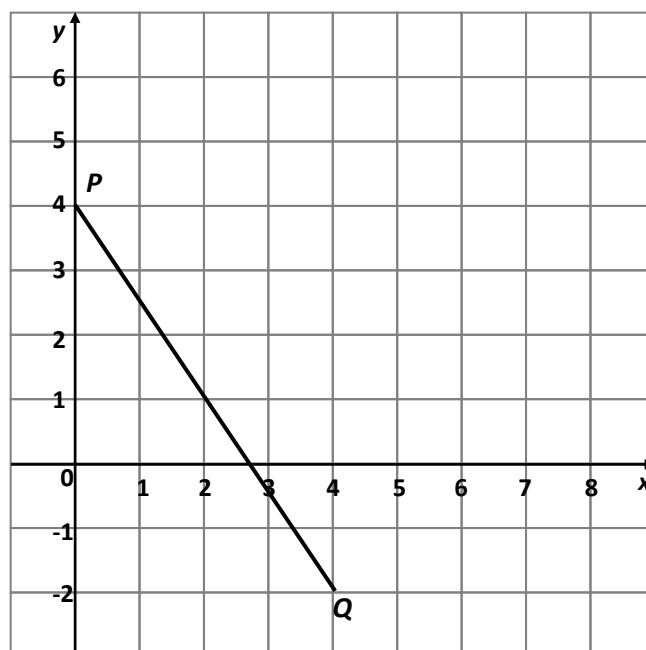
| | | | | | | |
|---|---|---|---|---|--|---|
| Number of boxes (b) | 1 | 2 | 3 | 4 | | 8 |
| Number of legs (l) | 4 | 6 | | | | |

b) Write down a formula for the number of legs (l) when you know the number of boxes (b).

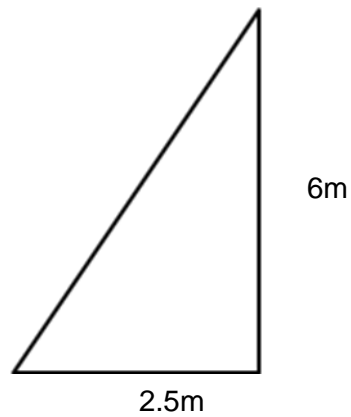
c) If 24 legs are used to build a storage unit, how many boxes are needed?

Calculating the gradient of a straight line from horizontal and vertical distances

1) The line PQ is drawn on the grid below. Calculate the gradient of the line PQ .

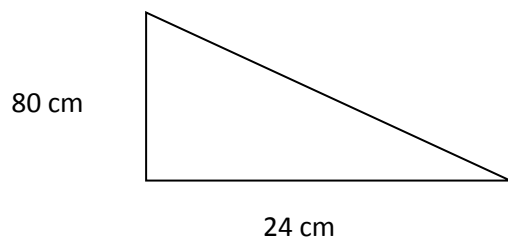


- 2) Calculate the gradient of the skate ramp.

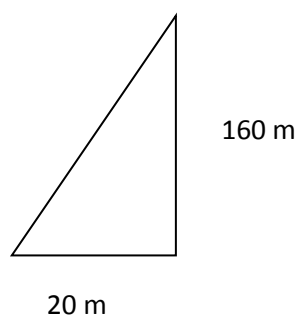


- 3) It is only safe to run on the treadmill when the gradient is less than 3.5.

Sarah wishes to run at the following settings.
Is it safe to do so?



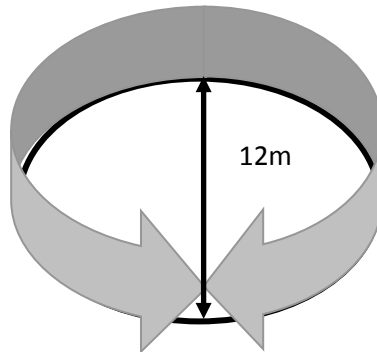
- 4) The Ski slope opposite is used to gather pace before jumping. Using the picture below calculate the gradient of the slope



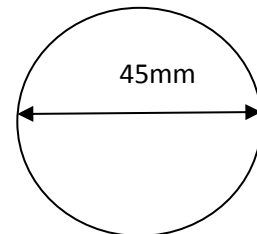
1.2 Applying geometric skills to circumference, area and volume

Calculating the circumference and area of a circle.

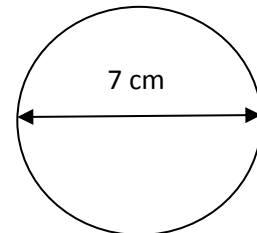
- 1) In the World's Strongest Man contest the men have to carry a heavy weight around the edge of a circle with a diameter of 12m



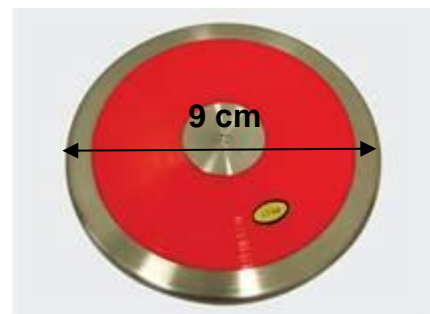
- a) Calculate the circumference of the circle
- b) Calculate the area of the circle.
- 2) A new coin is being designed to celebrate the birth of Prince William and Kate's baby being born. It has a diameter of 45mm
- a) Calculate the circumference of the coin
- b) Calculate the area coins face



- 3) This diagram shows the base of a cup.
The base of the cup is a circle with diameter of 7cm
- a) Calculate the circumference of the cup
- b) Calculate the area of the base.

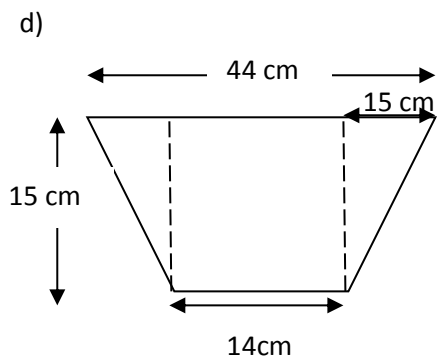
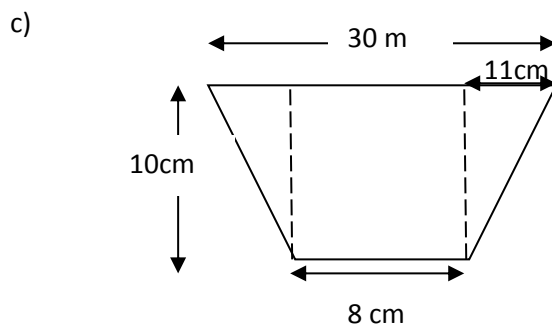
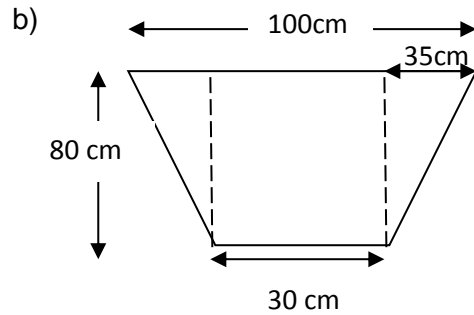
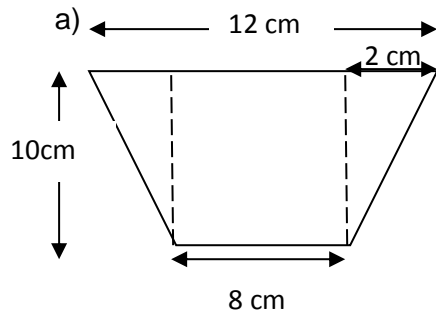


- 4) Shown is a discus with a diameter of 9cm
- a) Calculate the circumference of the discus
- b) Calculate the area of the discus.

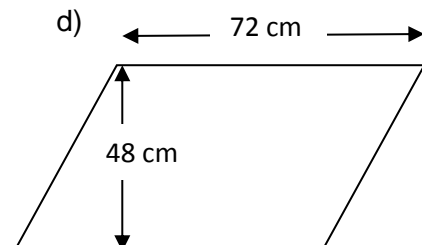
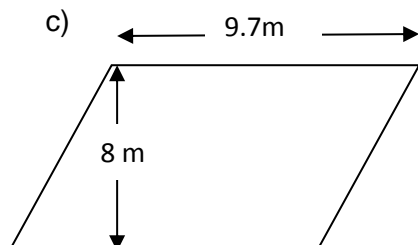
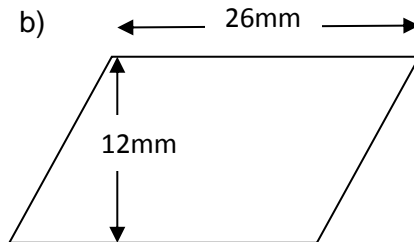
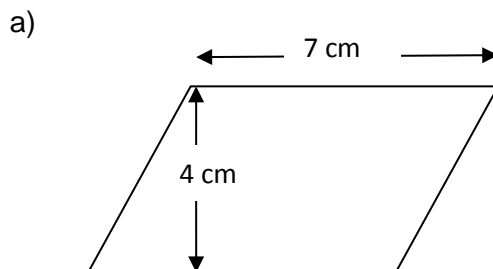


Calculating the area of a parallelogram, kite, trapezium.

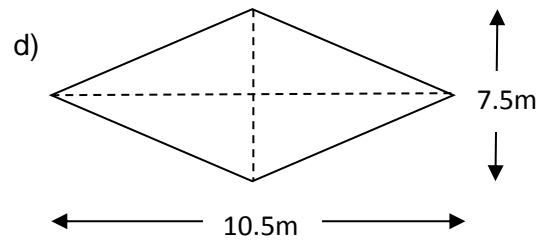
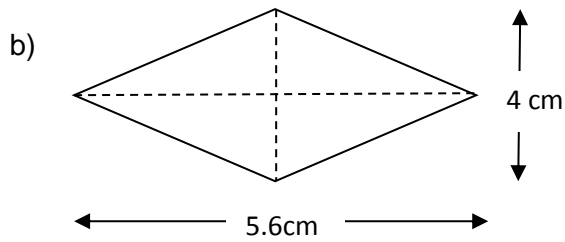
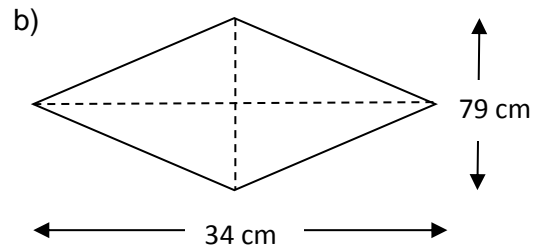
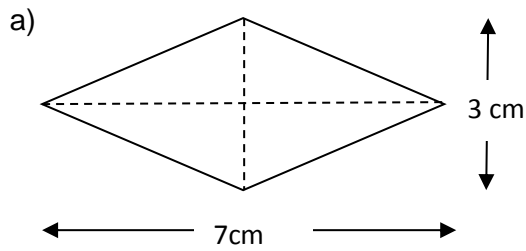
1) Calculate the area of the following:



2) Calculate the area of the following:



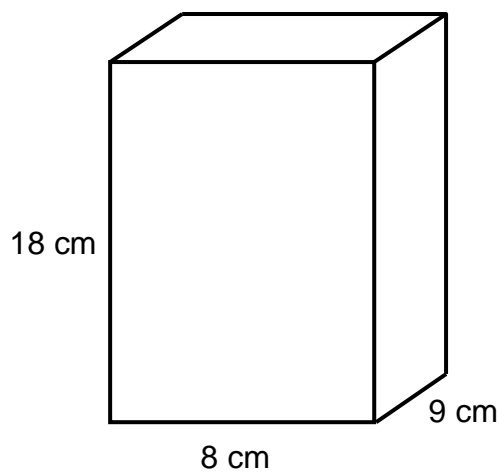
3) Calculate the area of the following:



Investigating the surface of a prism.

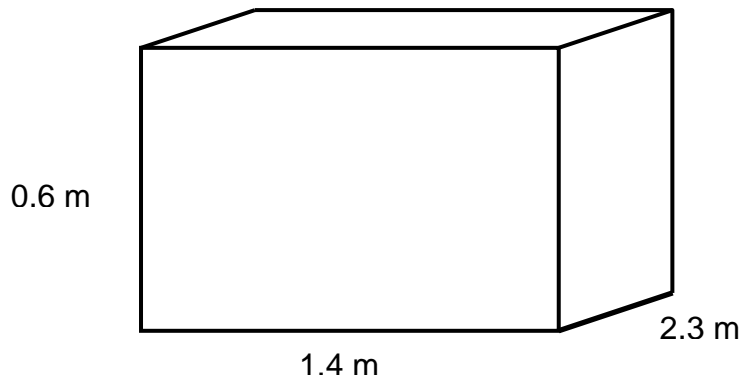
1) A cereal box is in the shape of a cuboid.

The cuboid is 9 centimetres long, 8 centimetres wide and 18 centimetres high, as shown in the diagram below.

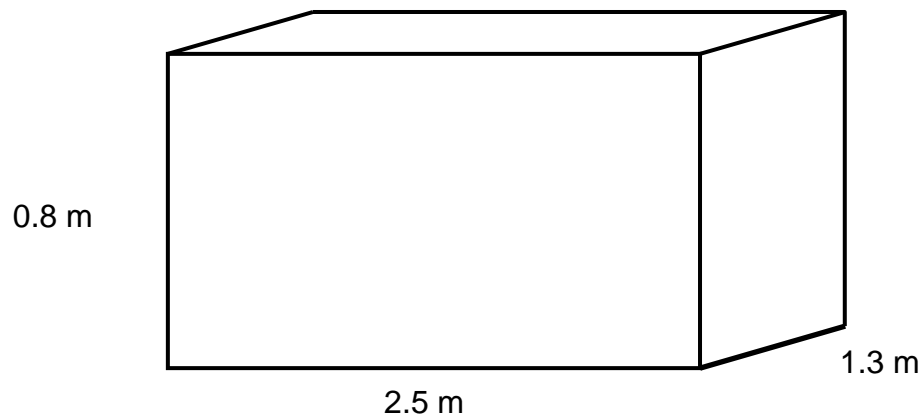


Calculate the surface area.

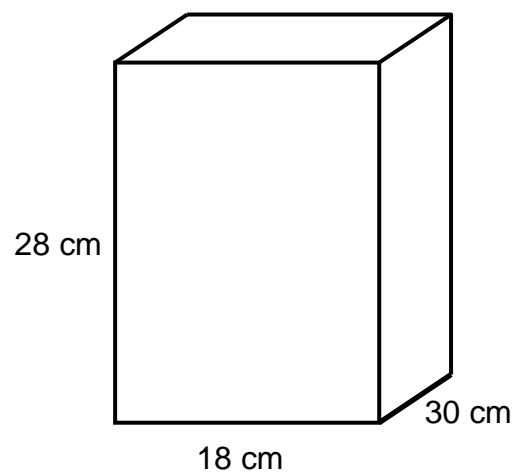
- 2) The diagram shows the frame for a rabbit hutch.
Calculate the surface area.



- 3) A set of soft toys in the shape of cuboids are to be made for a children's soft play area. Calculate the surface area of one cuboid.



- 4) A wooden box is made in the shape of a cuboid. Calculate the area of wood needed to make the box.

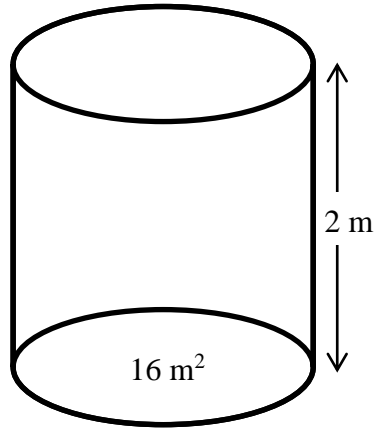


Calculating the volume of a prism.

1) A farmer has a large cylindrical container for storing grain to feed his hens.

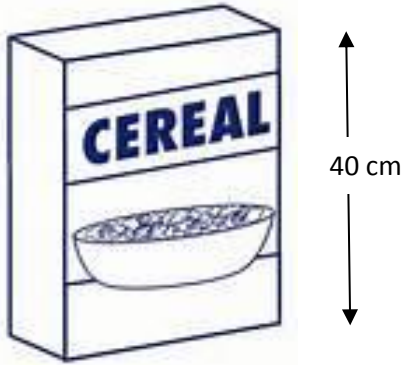
The area of the base of the container is 16 square metres.

The height of the container is 2 metres.



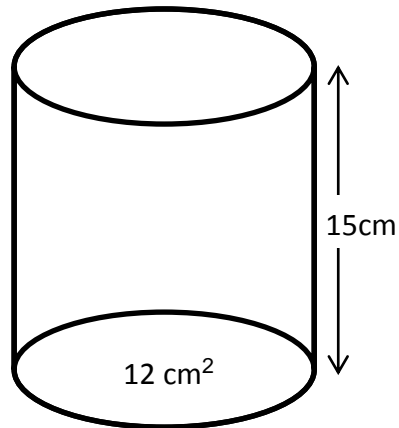
Calculate the volume of the container.

2) The cereal box below has been designed. Calculate the volume of cereal the box can hold. If the area of the base of the container is 108 square centimetres. The height of the container is 40 centimetres.



3) A new can has been designed for a sports drink. Calculate the volume of the can if the area of the base of the container is 12 squared centimetres.

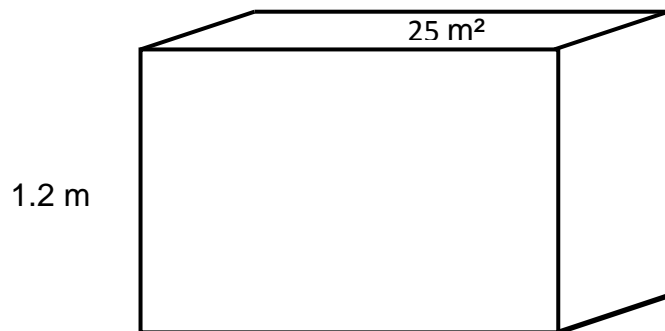
The height of the container is 15 centimetres.



4) Scott buys a box to keep all his tools in.

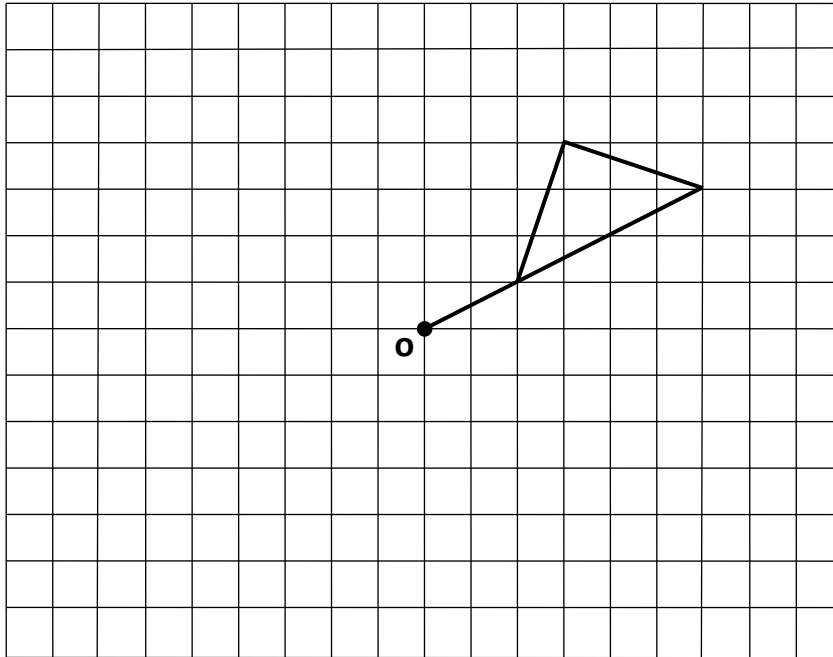
The area of the base of the container is 25 square metres.

The height of the container is 1.2 metres.

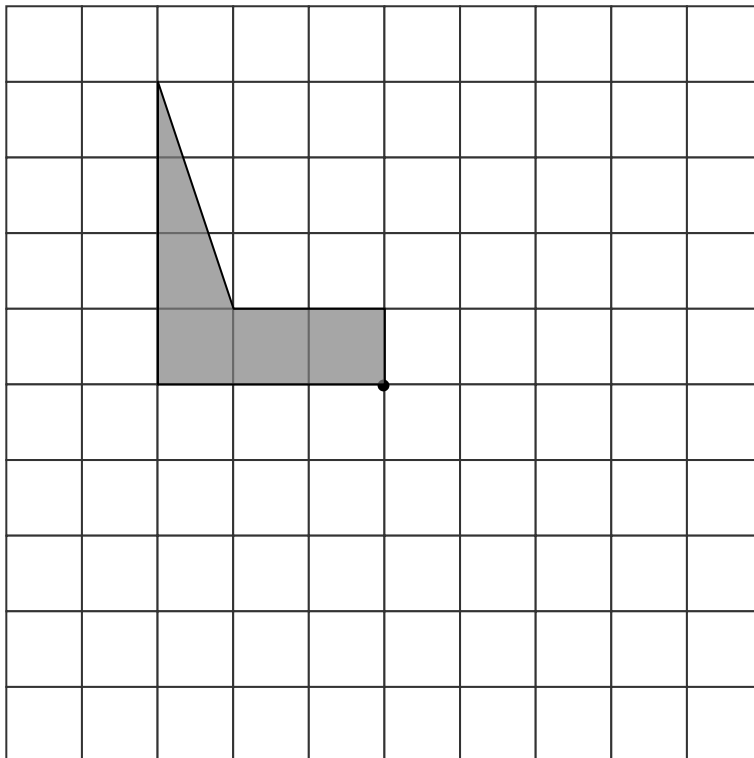


Using Rotational Symmetry

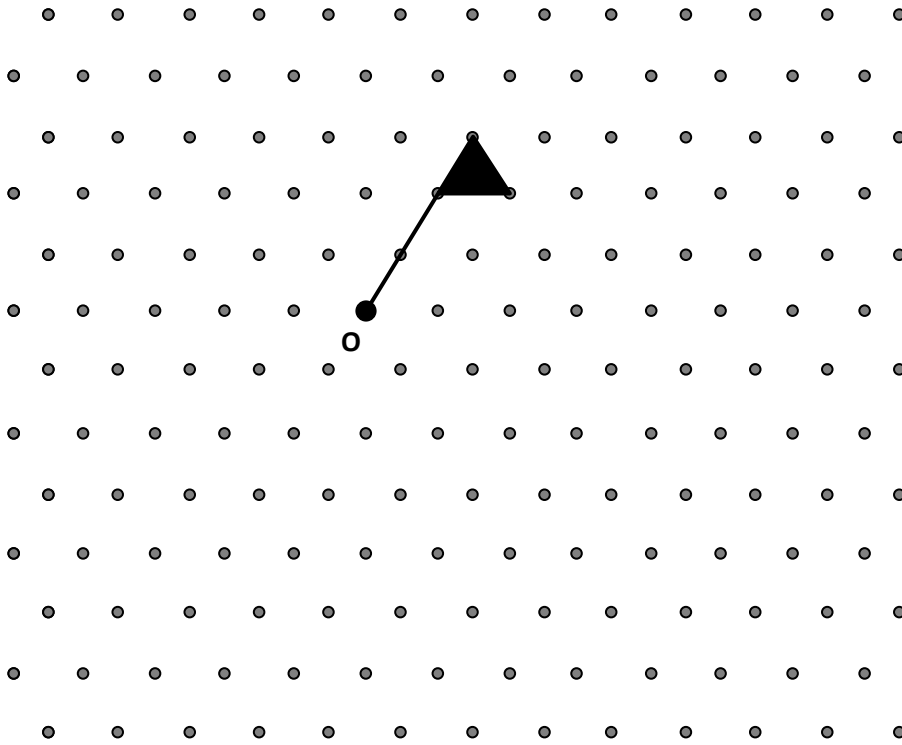
1) Copy and complete the diagram so that it has rotational symmetry of order 4 about the point **O**.



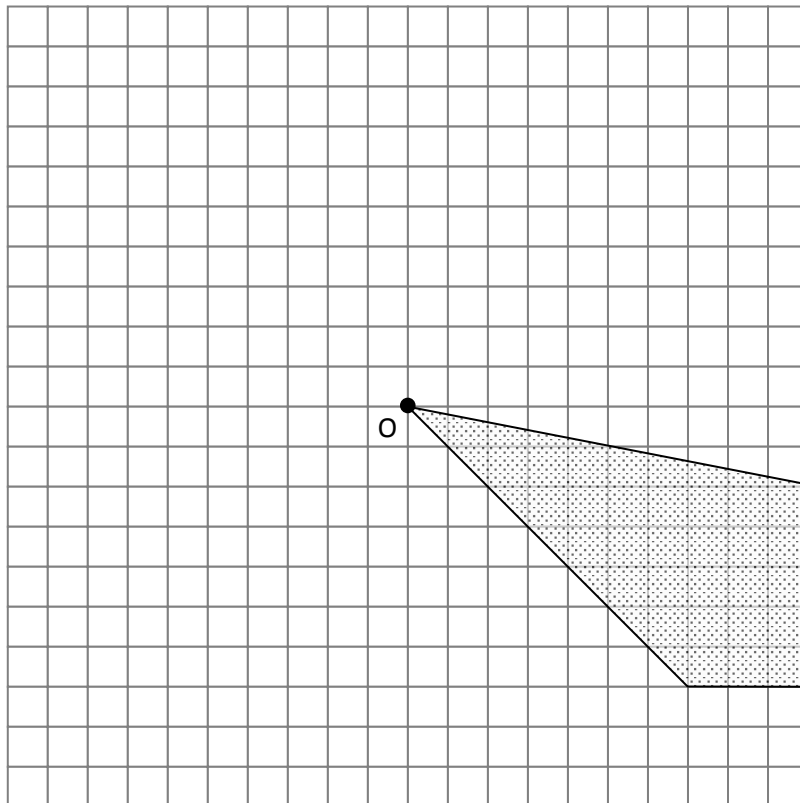
2) Copy and complete the shape below so that it has half-turn symmetry about **O**.



3) Copy and complete the diagram below so that it has rotational symmetry of order 3 about the point **O**.



4) Copy and complete the diagram so that the shape has half – turn symmetry about the point O



Applying statistical skills to representing and analysing data and to probability

Constructing a frequency table with class intervals from raw data.

- 1) A class did a survey on how many pencils each pupils had with them in school. The results of the survey are:

| | | | | | | |
|----|----|----|----|----|----|----|
| 4 | 7 | 2 | 18 | 1 | 16 | 19 |
| 15 | 13 | 0 | 9 | 17 | 4 | 6 |
| 10 | 12 | 15 | 8 | 3 | 14 | 19 |
| 14 | 15 | 18 | 5 | 16 | 3 | 6 |
| 5 | 18 | 12 | | | | |

Complete this frequency table for these results

| Score | Tally | Frequency |
|-------|-------|-----------|
| 0-4 | | |
| | | |
| | | |
| | | |
| | | Total = |

- 2) A teacher asked her class: "how many hours a week of you spend on a computer?" She asked them to give a rounded figure in hours. This was their response:

| | | | | | | |
|----|----|----|----|----|----|----|
| 3 | 6 | 9 | 2 | 23 | 18 | 6 |
| 8 | 29 | 27 | 2 | 1 | 0 | 5 |
| 19 | 23 | 20 | 21 | 7 | 4 | 23 |
| 8 | 7 | 1 | 0 | 25 | 24 | 8 |
| 13 | 18 | 15 | 16 | | | |

Complete this frequency table for these results

| Score | Tally | Frequency |
|-------|-------|-----------|
| 0-9 | | |
| | | |
| | | |
| | | Total = |

Determining statistics of a data set; interpreting calculated statistics.

1)

Dumfries drama club have eleven members. Their ages were recorded as follows:

12, 14, 13, 15, 15, 16, 13, 17, 18, 13, 16

- a) Calculate the mean of their ages.
 - b) Calculate the range of their ages
 - c) Strannraer Drama club has a mean age of 13.5 and a range of 9. Make two statements comparing the data.
- 2) Paul rolled a dice ten times with these results:

1, 2, 1, 3, 5, 3, 4, 6, 3, 4

- a) Calculate the mean score
- b) Calculate the modal score
- c) Karen also rolled a dice ten times
 - Her mean score was 3.2
 - Her modal score was 2

Make a valid statements comparing the data

3) Seven girls in S4 were weighed to the nearest kg. here are the results:

58kg, 62 kg, 44kg, 45kg, 54kg, 52kg, 56kg,

- a) Calculate the mean weight of the girls
- b) Calculate the range of the girls weight
- c) The same girls were weighed in S6
 - Their mean weight had change to 64.5kg
 - Their range had remained the same

Make two valid statements comparing the data.

- 4) The height of the first team Queen of the South team was taken at the start of the season. Here are the results:

1.68m, 1.77m, 1.53m, 1.77m, 1.69m, 1.71m, 1.76m, 1.67m, 1.70m, 1.63m, 1.80m

- Calculate the mean height of the team.
 - Calculate the range of the data
 - Calculate the modal height
- d) Annan also took the same data for their first team.
- the mean height was 1.76m
 - the modal height was 1.69m

Make two valid statements comparing the team's data.

Representing raw data in a pie chart.

- 1) Scott is a gardener and has to write a report for the estate he works for. In one particular part, he has sampled the patch got weeds so that he can order weed killer.

The table below shows the results.

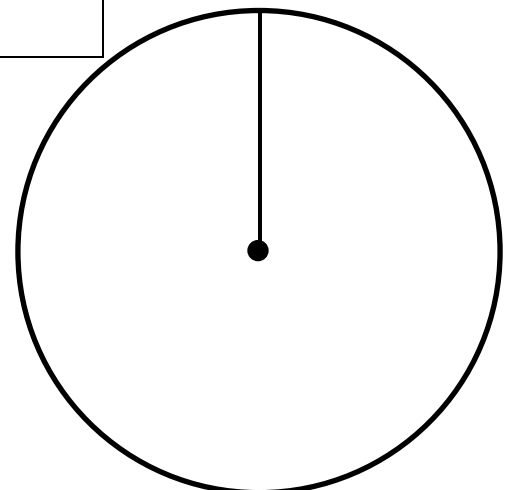
You have to complete the blank pie chart to show this same information

| Weed | Frequency |
|-----------|-----------|
| Buttercup | 6 |
| Daisy | 9 |
| Dandelion | 12 |
| Speedwell | 9 |

To help you complete the pie chart, fill in the blanks in the table.

| Weed | Frequency | Angle at the centre |
|-----------|-----------|---------------------|
| Buttercup | 6 | |
| Daisy | 9 | |
| Dandelion | 12 | |
| Speedwell | 9 | |

Now complete the pie chart.



- 2) Lucy plays for the school basketball team. She wants to improve her game so she analysed the type of passes she was able to make throughout a game.

The table below shows the results.

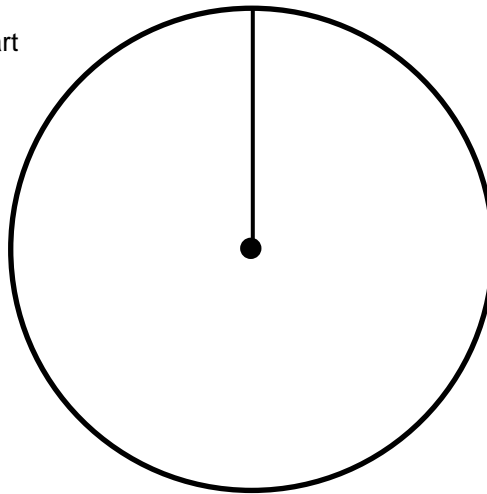
You have to complete the blank pie chart to show this same information

| Pass | Frequency |
|----------|-----------|
| Bounce | 20 |
| Chest | 10 |
| Overhead | 30 |

To help you complete the pie chart, fill in the blanks in the table.

| Pass | Frequency | Angle at the centre |
|----------|-----------|---------------------|
| Bounce | 20 | |
| Chest | 10 | |
| Overhead | 30 | |

Now complete the pie chart



- 3) The school wildlife club were looking to buy new bird feeders. To ensure they bought the most appropriate size they monitored the birds in the area for a week.

The table below shows the results.

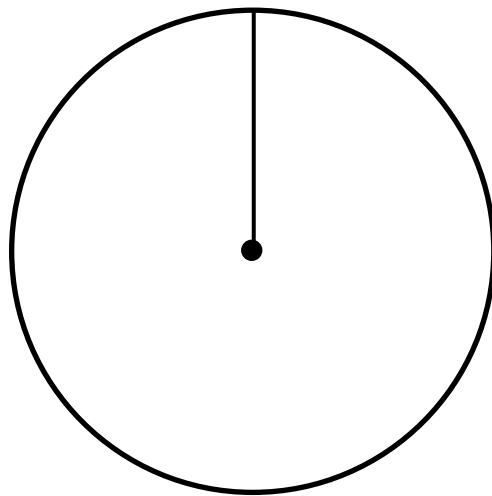
You have to complete the blank pie chart to show this same information

| Birds | Frequency |
|------------|-----------|
| Black bird | 35 |
| Chaffinch | 35 |
| Sparrow | 30 |

To help you complete the pie chart, fill in the blanks in the table.

| Birds | Frequency | Angle at the centre |
|------------|-----------|---------------------|
| Black bird | 35 | |
| Chaffinch | 35 | |
| Sparrow | 30 | |

Now complete the pie chart



- 4) Reg Wheels is opening a new car show room to help with sales they research the most popular colour of car

The table below shows the results.

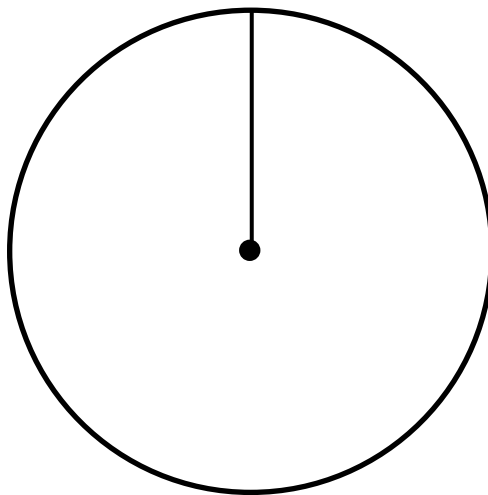
You have to complete the blank pie chart to show this same information

| Colour | Frequency |
|--------|-----------|
| Red | 35 |
| Black | 60 |
| White | 5 |

To help you complete the pie chart, fill in the blanks in the table.

| Colour | Frequency | Angle at the centre |
|--------|-----------|---------------------|
| Red | 35 | |
| Black | 60 | |
| White | 5 | |

Now complete the pie chart

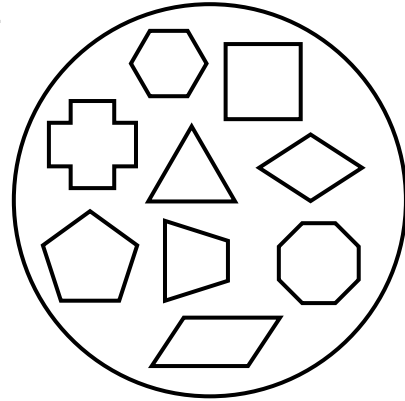


Using probability.

- 1) The diagram shows the lid of a child's toy. The child has to fit shapes through the differently shaped holes and drop them in a bucket.

If the child chooses a shape at random, what is the probability that it is a shape that has 5 sides or less?

Give your answer as a fraction in its simplest form.



- 2) A dice is rolled during a board game what is the probability that it lands on a prime number?



- 3) A bag of sweets contains 3 yellow, 4 blue and 2 red sweets what is the probability a yellow is picked out?



- 4) A fair ground game has a 4 by 4 grid shown opposite.

Coins are tossed at the board if it lands on a square you win whatever is in that square.

What is the probability that you win a prize?

| | | | |
|------|------|------|------|
| Lose | 20p | Lose | £1 |
| Lose | 10p | Lose | £1 |
| £1 | Lose | 30p | Lose |
| Lose | 50p | Lose | £1 |

Answers to 1.1 Applying algebraic skills to manipulating expressions and working with formulae

Using the distributive law in an expression with a numerical common factor to produce a sum of terms.

- 1) a) $6x-6$ b) $20y+8$ c) $12c-6$ d) $20a-25$
 2) a) $3y+1$ b) $3a+7$ c) $9n-7$ d) $3+2y$

Factorising a sum of terms with a numerical common factor.

- 1) a) $3(a+6)$ b) $5(y+2)$ c) $7(r-1)$ d) $9(9-5y)$

Simplifying an expression which has more than one variable.

- 1) a) $4x+2y$ b) $5p+4q$ c) $4g+5h$ d) $x+y+7$

Evaluating an expression or a formula which has more than one variable.

- 1) a) 24 b) 9 c) 18 d) 14
 2) a) i) 900 ii) 1800 b) i) £55 ii) £45 c) 94 units² d) 6

Extending a straightforward number or diagrammatic pattern and determining its formula.

1)

a)

| | | | | |
|----------------------------------|---|---|----|----|
| Number of vertical posts (v) | 2 | 3 | 4 | 5 |
| Number of barrier posts (b) | 4 | 8 | 12 | 16 |

b) $b=4v-4$

c) i) 31

ii) 120

2)

a)

| | | | | | |
|----------------------------------|---|---|---|--|----|
| Number of bars (<i>b</i>) | 2 | 3 | 4 | | 8 |
| Number of chains (<i>c</i>) | 3 | 6 | 9 | | 21 |

b) $C=3b-3$

c) 17

3)

a)

| | | | | | | | |
|-------------------------------|---|---|---|----|----|--|----|
| Number of bars (<i>b</i>) | 2 | 3 | 4 | 5 | 6 | | 10 |
| Number of chains (<i>c</i>) | 3 | 6 | 9 | 12 | 15 | | 27 |

b) $C=3b-3$

c) $b=22$

4)

a)

| | | | | | | |
|------------------------------|---|---|---|----|--|----|
| Number of boxes (<i>b</i>) | 1 | 2 | 3 | 4 | | 8 |
| Number of legs (<i>l</i>) | 4 | 6 | 8 | 10 | | 18 |

b) $l=2b+2$

c) $b=11$

Calculating the gradient of a straight line from horizontal and vertical distances

1) $m = \frac{3}{2}$

2) $m = \frac{12}{5}$

3) $m=3.33$ which is less than the safety regulations therefore it is safe to run

4) $m = 80$

Answers to 1.2 Applying geometric skills to circumference, area and volume

Calculating the circumference and area of a circle.

- 1) a) 37.7m b) 113.09m²
- 2) a) 141.4mm b) 1590.4mm²
- 3) a) 21.99cm b) 38.48cm²
- 4) a) 28.3cm b) 63.6cm²

Calculating the area of a parallelogram, kite, trapezium.

- 1) a) 120cm² b) 8000cm² c) 300cm² d) 660cm²
- 2) a) 28cm² b) 312cm² c) 77.6m² d) 3456m²
- 3) a) 10.5cm² b) 1343cm² c) 11.2cm² d) 39.375m²

Investigating the surface of a prism.

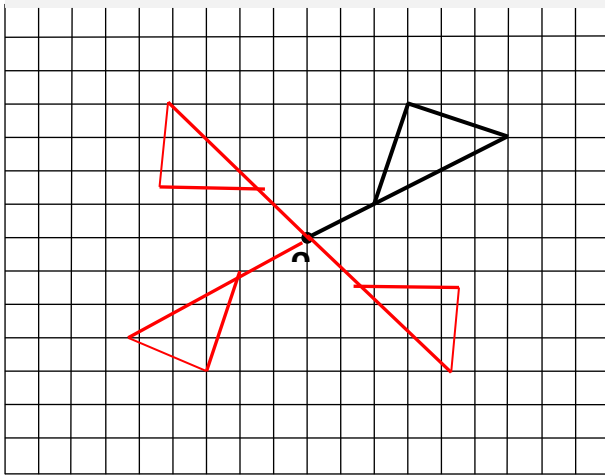
- 1) 756cm²
- 2) 10.88 cm²
- 3) 12.58 cm²
- 4) 12.58 cm²

Calculating the volume of a prism.

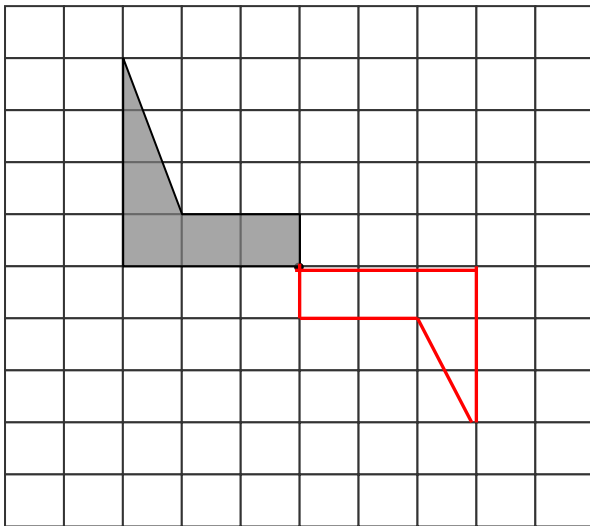
- 1) 32m³
- 2) 4320cm³
- 3) 180cm³
- 4) 30m³

Using Rotational Symmetry

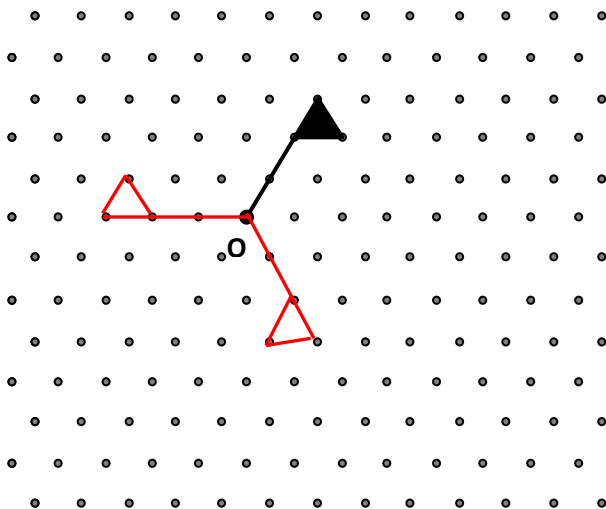
1)



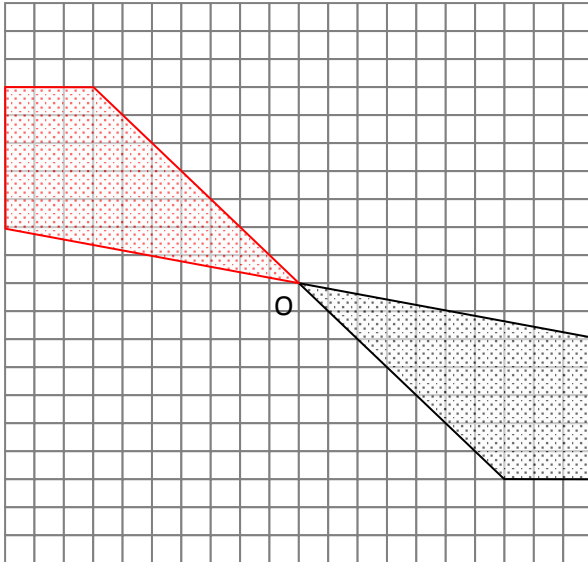
2)



3)



4)



Answers to 1.3 Applying statistical skills to representing and analysing data and to probability

1)

| Score | Tally | Frequency |
|-------|-------|------------|
| 0-4 | | 7 |
| 5-9 | | 7 |
| 10-14 | | 6 |
| 15-19 | | 11 |
| | | Total = 31 |

2)

| Score | Tally | Frequency |
|-------|-------|------------|
| 0-9 | | 17 |
| 10-19 | | 6 |
| 20-29 | | 9 |
| | | Total = 32 |

3)

| Score | Tally | Frequency |
|-------|-------|------------|
| 20-29 | | 3 |
| 30-39 | | 3 |
| 0-49 | | 7 |
| 50-59 | | 3 |
| 60-69 | | 3 |
| 70-79 | | 1 |
| | | Total = 20 |

4)

| Score | Tally | Frequency |
|-------|-------|------------|
| 40-49 | | 2 |
| 50-59 | | 2 |
| 60-69 | | 4 |
| 70-79 | | 9 |
| 80-89 | | 8 |
| 90-99 | | 3 |
| | | Total = 28 |

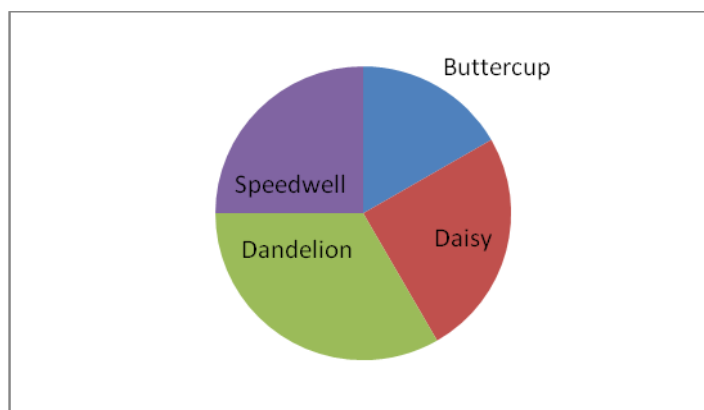
Determining statistics of a data set; interpreting calculated statistics.

- 1) a) 14.73 b) 6
 c) The average age attending Stranraer is lower than Dumfries.
 Stranraer has a wider spread of ages attending
- 2) a) 3.2 b) 3
 c) The two dice gave the same mean average.
- 3) a) 53 b) 18
 c) The average weight of the girls as increased.
 The spread of their weight has remained the same.
- 4) a) 1.70 b) 0.27 c) 1.77
 d) The average players for Annan is taller than QoS
 Annan have a few smaller players.

Representing raw data in a pie chart.

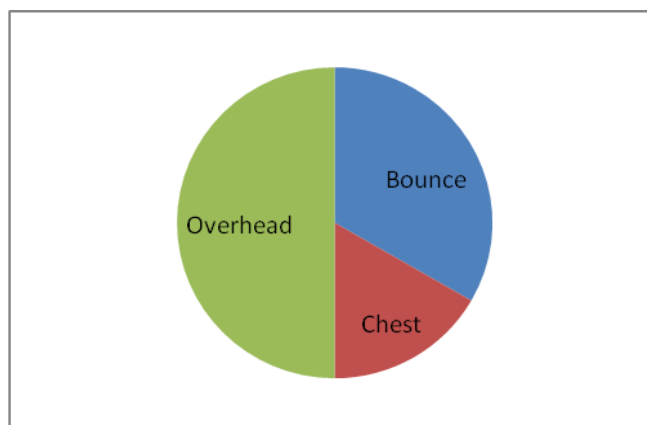
1)

| Weed | Frequency | Angle at the centre |
|-----------|-----------|---------------------|
| Buttercup | 6 | 60° |
| Daisy | 9 | 90° |
| Dandelion | 12 | 120° |
| Speedwell | 9 | 90° |



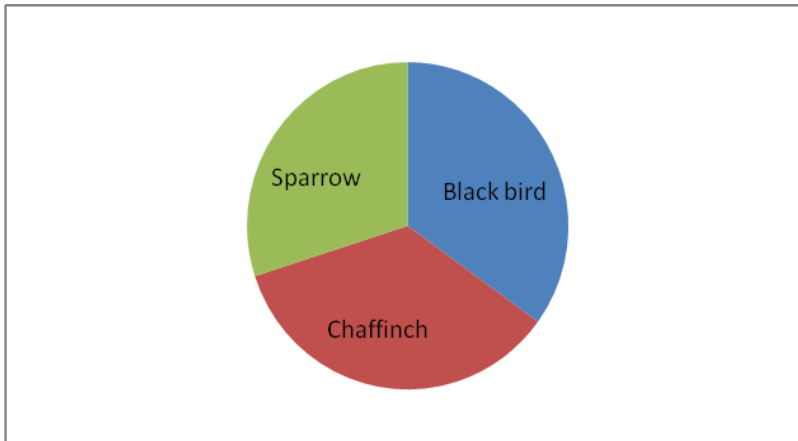
2)

| Pass | Frequency | Angle at the centre |
|----------|-----------|---------------------|
| Bounce | 20 | 120° |
| Chest | 10 | 60° |
| Overhead | 30 | 180° |



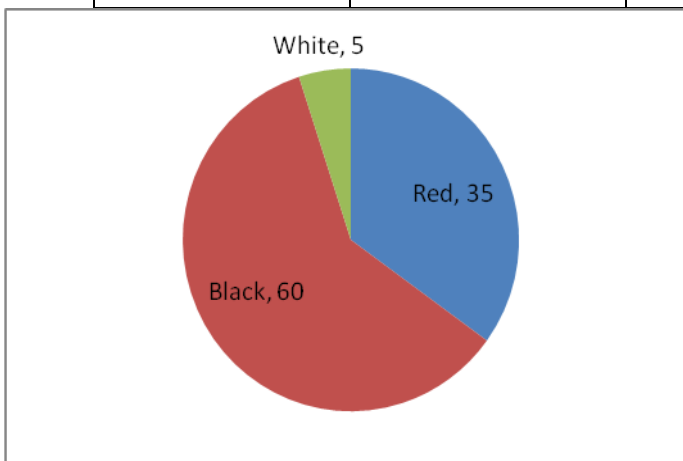
3)

| Birds | Frequency | Angle at the centre |
|------------|-----------|---------------------|
| Black bird | 35 | 126 |
| Chaffinch | 35 | 126 |
| Sparrow | 30 | 108 |



4)

| Colour | Frequency | Angle at the centre |
|--------|-----------|---------------------|
| Red | 35 | 126° |
| Black | 60 | 216° |
| White | 5 | 18° |



Using probability.

1) $\frac{2}{3}$

$\frac{1}{2}$

2) $\frac{1}{3}$

$\frac{1}{2}$

3) $\frac{1}{3}$

4) $\frac{1}{2}$