

(a) $f(x) = 2x + 1$, $g(x) = x^2 + k$, where k is a constant.

(i) Find $g(f(x))$; 2

(ii) find $f(g(x))$.

(b) (i) Show that the equation $g(f(x)) - f(g(x)) = 0$ simplifies to $2x^2 + 4x - k = 0$.

(ii) Determine the nature of the roots of this equation when $k = 6$.

(iii) Find the value of k for which $2x^2 + 4x - k = 0$ has equal roots. 7

Two sequences are defined by the recurrence relations

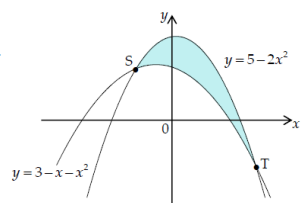
$$u_{n+1} = 0.2u_n + p, \quad u_0 = 1$$

$$v_{n+1} = 0.6v_n + q, \quad v_0 = 1$$

If both sequences have the same limit, express p in terms of q . 4

Dec 17-10:42

Part of the graphs of $y = 3 - x - x^2$ and $y = 5 - 2x^2$ are shown opposite.



The curves intersect at the points S and T.

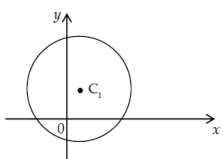
(a) Find the coordinates of S and T. 4

(b) Find the shaded area enclosed between the two curves. 5

Dec 17-10:43

A circle with centre C_1 has equation $x^2 + y^2 - 2x - 6y - 15 = 0$.

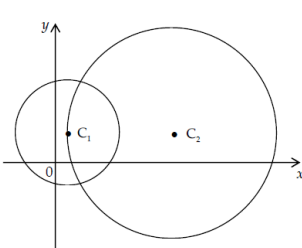
(a) Write down the coordinates of the centre and calculate the length of the radius of this circle. 2



A second circle with centre C_2 has a diameter twice that of the circle with centre C_1 .

C_1 lies on the circumference of this second circle.

The line joining C_1 and C_2 is parallel to the x -axis.



State the equation of the circle with centre C_2

Dec 17-10:44

Find $\int (x-3)(3x+1) dx$.

Given that $f(x) = 2x^4 - 5x$, find $f'(2)$.

Find all the values of x in the interval $0 \leq x < 2\pi$ for which $\cos x = -\frac{\sqrt{3}}{2}$.

Dec 17-10:51

Find the values of x for which the function $f(x) = 5 + 24x + 3x^2 - x^3$ is decreasing. 5

A circle, centre C , has equation $x^2 + y^2 - 4x - 2y - 20 = 0$.

(a) Find the centre C and radius of this circle. 2

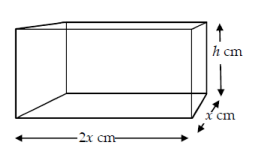
(b) (i) Show that the point $P(5, -3)$ lies on the circumference of the circle. 4

(ii) Find the equation of radius CP . 4

(c) Find the equation of the chord which passes through $(7, 1)$ and is perpendicular to radius CP . 3

Dec 17-10:54

A closed wooden box, in the shape of a cuboid, is constructed from a sheet of wood of area 600 cm^2 .



The base of the box measures $2x \text{ cm}$ by $x \text{ cm}$.

The height of the box is $h \text{ cm}$.

(a) Assuming the thickness of the sides of the box are negligible, show that the volume (in cubic centimetres) of the box is given by

$$V(x) = 200x - \frac{4}{3}x^3$$
 3

(b) (i) Calculate the value of x for which this volume is a maximum.

(ii) Find the maximum volume of the box. 7

Dec 17-10:55

24. The diagram shows part of the quartic with equation $y = g(x)$.
 There are stationary points at $x = -2$, $x = 0$ and $x = a$.

On separate diagrams sketch the graph of

(a) $y = g'(x)$. 3

(b) $y = g'(x-3)$. 2

Dec 17-10:54

A line joins the points $P(-4, 3)$ and $Q(2, -7)$.

Find the equation of the perpendicular bisector of PQ . 4

Show that the line with equation $y = 2x + 10$ is a tangent to the circle with equation $x^2 + y^2 - 2x - 4y - 15 = 0$ and find the coordinates of the point of contact of the tangent and circle. 6

Dec 17-10:58

A function f is defined by $f(x) = x^3 - 2x^2 - 4x + 1$, where $0 \leq x \leq 3$.
 Find the maximum and minimum values of f . 5

The graph of the cubic function $y = f(x)$ is shown in the diagram. There are turning points at $(1, 1)$ and $(3, 5)$.
 Sketch the graph of $y = f'(x)$. 3

Dec 17-10:59

A new '24 hour anti-biotic' is being tested on a patient in hospital.
 It is known, that over a 24 hour period, the amount of anti-biotic remaining in the bloodstream is reduced by 80%.
 On the first day of the trial, an initial 250 mg dose is given to a patient at 7 a.m.

(a) After 24 hours and just prior to the second dose being given, how much anti-biotic remains in the patient's bloodstream? 1

The patient is then given a further 250 mg dose at 7 a.m. and at this time each subsequent morning thereafter.

(b) A recurrence relation of the form $u_{n+1} = au_n + b$ can be used to model this course of treatment.
 Write down the values of a and b . 2

It is also known that more than 350 mg of the drug in the bloodstream results in unpleasant side effects.

(c) Is it safe to administer this anti-biotic over an extended period of time? 4

Dec 17-11:00