

Recurrence Relations HW Solutions

Q1. Initial amount = £17000 payments = £3000 p.a Interest = 4.02% p.a

(a) $U_{n+1} = 1.0402U_n - 3000$ (2)

(b) $U_0 = 17000$

$U_1 = 17000(1.0402) - 3000 = £14683.40$

$U_2 = 14683.4(1.0402) - 3000 = £12273.67$

$U_3 = 12273.67(1.0402) - 3000 = £9767.07$

$U_4 = 9767.07(1.0402) - 3000 = £7159.71$ (2)

Q2. (a) $U_{n+1} = 0.85U_n + 180$ (2)

(b) $L = \frac{b}{1-a} = \frac{180}{1-0.85} = 1200$

$1000 < 1200 < 1500 \Rightarrow$ the safety of the species will be guaranteed. (3)

Q3. $U_{n+1} = 0.6U_n + 10$ $U_2 = 52$

(a) Find U_0

$U_2 = 0.6U_1 + 10$

$52 = 0.6U_1 + 10$ ✓

$42 = 0.6U_1$

$U_1 = 70$ ✓

$U_1 = 0.6U_0 + 10$

$70 = 0.6U_0 + 10$ ✓

$60 = 0.6U_0$

$U_0 = 100$ ✓ (4)

(b) This sequence has a limit because $-1 < a < 1$ for a limit to exist and $-1 < 0.6 < 1$ ✓

$L = \frac{b}{1-a} = \frac{10}{1-0.6} = 25$ ✓ (3)

Q4. $U_{n+1} = 0.8U_n + 12$ and $V_{n+1} = aV_n + 18$ have the same limit, find a

$L = \frac{b}{1-a} = \frac{12}{1-0.8} = 60$ ✓

$L = \frac{b}{1-a} = \frac{18}{1-a} = 60$ ✓

$18 = 60(1-a) \Rightarrow 18 = 60 - 60a$

$-42 = -60a$

$a = 0.7$ ✓

(3)

Q5. $U_{n+1} = pU_n + q$ $-1 < p < 1$ $U_0 = 12$

(a) $U_1 = 15$ $U_2 = 16$, find p and q

$U_1 = pU_0 + q$

$15 = 12p + q$

$U_2 = pU_1 + q$

$16 = p(15) + q$

✓ Interpret sequence

$12p + q = 15$

⊖ $15p + q = 16$

$-3p = -1$

$p = \frac{1}{3}$ ✓ solve for one variable

$15 = 12p + q$

$15 = 4 + q$

$q = 11$ ✓

✓ solve for 2nd variable

③

(b) $U_{n+1} = \frac{1}{3}U_n + 11$

$L = \frac{b}{1-a} = \frac{11}{1-\frac{1}{3}} = 16.5$ ✓ answer

✓ knowing how to find limit

②

Q6. (b) $U_{n+1} = 0.75U_n + 1$ ✓ ①

(b) (i) $L = \frac{b}{1-a} = \frac{1}{1-0.75} = 4$ ✓ yes it is safe because the maximum level is 4g. ✓ ③

(a)

$U_0 = 0$

$U_1 = 0.75(0) + 1 = 1$

$U_2 = 0.75(1) + 1 = 1.75$

$U_3 = 0.75(1.75) + 1 = 2.3125$

$U_4 = 0.75(2.3125) + 1 = 2.734375$

It would take 4 feeds

②

Total = 30 marks