(*a*) For $y = \frac{5x+1}{x^2+2}$, find $\frac{dy}{dx}$. Express your answer as a single, simplified fraction. **3**

$$dy/dx = \frac{(x^2+2)(5) - (5x+1)(2x)}{(x^2+2)^2}$$

$$= \frac{5x^2+10-10x^2-2x}{(x^2+2)^2}$$

$$= \frac{10-5x^2-2x}{(x^2+2)^2}$$

Differentiate $f(x) = e^{\cos x}\sin^2 x$. **3**

$$f'(x) = e^{\cos x}(2\sin x\cos x) + \sin^2 x(-\sin x\, e^{\cos x})$$

$$= 2e^{\cos x}\sin x\cos x - e^{\cos x}\sin^3 x$$

$$= e^{\cos x}\sin x(2\cos x - \sin^2 x)$$

(*a*) Given $f(x) = \frac{3x+1}{x^2+1}$, obtain $f'(x)$. **3**

$$f'(x) = \frac{(x^2+1)(3) - (3x+1)(2x)}{(x^2+1)^2}$$

$$= \frac{3x^2+3-6x^2-2x}{(x^2+1)^2}$$

$$= \frac{3-2x-3x^2}{(x^2+1)^2}$$

(*a*) Given

$$f(x) = \frac{x^2-1}{x^2+1},$$

obtain $f'(x)$ and simplify your answer. **3**

$$f'(x) = \frac{(x^2+1)(2x) - (x^2-1)(2x)}{(x^2+1)^2}$$

$$= \frac{\cancel{2x^3} + 2x - \cancel{2x^3} + 2x}{(x^2+1)^2}$$

$$= \frac{4x}{(x^2+1)^2}$$

Given $f(x) = \sin x\cos^3 x$, obtain $f'(x)$. **3**

$$f'(x) = (\sin x)(3\cos^2 x(-\sin x)) + \cos^3 x(\cos x)$$

$$= -3\cos^2 x\sin^2 x + \cos^4 x$$

$$= \cos^2 x(-3\sin^2 x + \cos^2 x)$$

$f(x) = \sqrt{x}\,e^{-x}$, find $f'(x)$

$$f(x) = x^{\frac{1}{2}}e^{-x}$$

$$f'(x) = x^{\frac{1}{2}}(-e^{-x}) + e^{-x}\left(\tfrac{1}{2}x^{-\frac{1}{2}}\right)$$

$$= -\sqrt{x}\,e^{-x} + \frac{1}{2\sqrt{x}}e^{-x}$$

$$= e^{-x}\left(-\sqrt{x} + \frac{1}{2\sqrt{x}}\right)$$

$$=$$

Trigonometric Functions

$\sin x^\circ = \frac{o}{h}$

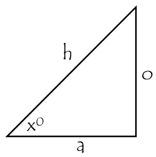
$\cos x^\circ = \frac{a}{h}$

$\tan x^\circ = \frac{o}{a}$

$\sec x^\circ = \frac{h}{a} = \frac{1}{\cos x}$

$\text{cosec}\, x^\circ = \frac{h}{o} = \frac{1}{\sin x}$

$\text{cotan}\, x^\circ = \frac{a}{o} = \frac{1}{\tan x}$



Differentiating Trigonometric Functions

$f(x) = \sin ax$ $\quad$ $f(x) = \cos ax$

$f'(x) = a\cos ax$ $\quad$ $f'(x) = -a\sin ax$

Differentiating Trigonometric Functions

Finding the derivative of tanx

$\tan x = \frac{\sin x}{\cos x}$

$f(x) = \frac{\sin x}{\cos x}$

$f'(x) = \frac{\cos x(\cos x) - \sin x(-\sin x)}{\cos^2 x}$

$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} = \sec^2 x$

Differentiating Trigonometric Functions

Finding the derivative of secx

$\sec x = \frac{1}{\cos x}$

$f(x) = \frac{1}{\cos x} = (\cos x)^{-1}$

$f'(x) = -1(\cos x)^{-2}(-\sin x)$

$= \frac{\sin x}{\cos^2 x} = \tan x \cdot \frac{1}{\cos x} = \tan x \sec x$

Differentiating Trigonometric Functions

Finding the derivative of cosecx

$\text{cosec}\, x = \frac{1}{\sin x}$

$f(x) = (\sin x)^{-1}$

$f'(x) = -1(\sin x)^{-2}(\cos x)$

$= \frac{-\cos x}{\sin^2 x} = \frac{-\cos x}{\sin x} \cdot \frac{1}{\sin x}$

$= -\frac{1}{\tan x} \cdot \text{cosec}\, x$

$= -\cot x \,\text{cosec}\, x$

Differentiating Trigonometric Functions

Finding the derivative of cotx

$f(x) = \frac{1}{\tan x} = \frac{\cos x}{\sin x}$

$f'(x) = \frac{\sin x(-\sin x) - \cos x(\cos x)}{\sin^2 x}$

$= \frac{-\sin^2 x - \cos^2 x}{\sin^2 x} = \frac{-1(\sin^2 x + \cos^2 x)}{\sin^2 x}$

$= \frac{-1}{\sin^2 x} = -\text{cosec}^2 x$

Differentiating Trigonometric Functions

Examples: Differentiate the following

1. y = tan3x

$\frac{dy}{dx} = 3\sec^2 3x$

Differentiating Trigonometric Functions

Examples: Differentiate the following

2. y = cot(x^4)

$\frac{dy}{dx} = -4x^3\text{cosec}^2(x^4)$

Differentiating Trigonometric Functions

Examples: Differentiate the following

3. y = $\cot^4 x$ $= (\cot x)^4$

$\frac{dy}{dx} = 4(\cot x)^3 . -\text{cosec}^2 x$

$-4\cot^3 x\,\text{cosec}^2 x$

Differentiating Trigonometric Functions

Examples: Differentiate the following

4. y = $\sec^2(3x - 1)$ $= (\sec(3x-1))^2$

$\frac{dy}{dx} = 2(\sec(3x-1)) . 3\sec(3x-1)\tan(3x-1)$

$= 6\sec^2(3x-1)\tan(3x-1)$

Differentiating Trigonometric Functions

Examples: Differentiate the following

5. y = $x^2\sin x + 8\cos^2 3x$ $= x^2\sin x + 8(\cos 3x)^2$

$\frac{dy}{dx} = x^2(\cos x) + \sin x . (2x) + 16(\cos 3x) . -3\sin 3x$

$= x^2\cos x + 2x\sin x - 48\cos 3x \sin 3x$

$= x^2\cos x + 2x\sin x - 24\sin 6x$

$\sin 2A = 2\sin A\cos A$
from Higher

Differentiating Trigonometric Functions

Examples: Differentiate the following

6. y = $\frac{\tan x}{x^2}$

$\frac{dy}{dx} = \frac{x^2(\sec^2 x) - \tan x . (2x)}{x^4}$

$= \frac{x^2\sec^2 x - 2x\tan x}{x^4}$

$\frac{x\sec^2 x - 2\tan x}{x^3}$

Differentiating Trigonometric Functions

Examples: Differentiate the following

7. y = ln(tanx)

$$\frac{dy}{dx} = \frac{1}{\tan x} \cdot \sec^2 x$$

$$= \frac{1}{\tan x} \cdot \frac{1}{\cos^2 x}$$

$$= \frac{1}{\frac{\sin x}{\cos x}} \cdot \frac{1}{\cos^2 x}$$

$$= \frac{\cos x}{\sin x} \cdot \frac{1}{\cos^2 x} = \frac{1}{\sin x \cos x}$$

$$y = \ln(2x)$$

$$\frac{dy}{dx} = \frac{1}{2x} \cdot 2$$

$$= \frac{2}{2x}$$

Differentiating Trigonometric Functions

Examples: Differentiate the following

8. y = secxtanx

$$\frac{dy}{dx} = \sec x(\sec^2 x) + \tan x(\sec x \tan x)$$

$$= \sec^3 x + \sec x \tan^2 x$$

$$= \sec x(\sec^2 x + \tan^2 x)$$

$$= \sec x\left(\frac{1}{\cos^2 x} + \frac{\sin^2 x}{\cos^2 x}\right)$$

$$= \sec x\left(\frac{1+\sin^2 x}{\cos^2 x}\right)$$

$$= \sec x\left(\frac{1+(1-\cos^2 x)}{\cos^2 x}\right)$$

$$= \sec x\left(\frac{2-\cos^2 x}{\cos^2 x}\right) = \sec x\left(\frac{2}{\cos^2 x} - \frac{\cos^2 x}{\cos^2 x}\right)$$

$$= \sec x(2\sec^2 x - 1)$$